

$$2 \sin \left( x + \frac{\pi}{3} \right) = 2 \left[ \sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} \right] \quad \text{ـ(1)}$$

$$= 2 \left( \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right)$$

$$= \sin x + \sqrt{3} \cos x$$

$$\sin x - \cos x = \sqrt{2} \left( \frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x \right) \quad \text{ـب}$$

$$= \sqrt{2} \left( \sin \left( x - \frac{\pi}{4} \right) \right)$$

إذن يكفي أن نأخذ  $a = \frac{\pi}{4}$

(2) نعتبر  $x \neq \frac{\pi}{4} + k\pi$  حيث  $k \in \mathbb{Z}$

$$(E) \Leftrightarrow \frac{\sin x}{\cos x} = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} - 1}$$

$$\Leftrightarrow (\sqrt{2} - 1) \sin x = \cos x (\sqrt{3} + \sqrt{2})$$

$$\Leftrightarrow \sqrt{2} (\sin x - \cos x) = \sin x + \sqrt{3} \cos x$$

$$\sin x - \cos x = \sqrt{2} \sin \left( x - \frac{\pi}{4} \right) \quad \text{وبما أن :}$$

$$2 \sin \left( x + \frac{\pi}{3} \right) = \sin x + \sqrt{3} \cos x \quad \text{و}$$

$$(E) \Leftrightarrow 2 \sin \left( x - \frac{\pi}{4} \right) = 2 \sin \left( x + \frac{\pi}{3} \right) \quad \text{فإن :}$$

$$\Leftrightarrow \sin \left( x - \frac{\pi}{4} \right) = \sin \left( x + \frac{\pi}{3} \right)$$

$$(E) \Leftrightarrow \sin \left( x - \frac{\pi}{4} \right) = \sin \left( x + \frac{\pi}{3} \right) \quad \text{ـ(3)}$$

$$\Leftrightarrow \begin{cases} x - \frac{\pi}{4} = x + \frac{\pi}{3} + 2k\pi; k \in \mathbb{Z} \\ x - \frac{\pi}{4} = \pi - x - \frac{\pi}{3} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow x = \frac{11\pi}{24} + k\pi$$

وبما أن  $x \in [0, 2\pi]$  فإن  $k \in \{0, 1\}$

$$S = \left\{ \frac{11\pi}{24}, \frac{35\pi}{24} \right\} \quad \text{وبالتالي فإن :}$$

$$\tan x < \frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} - 1} \Leftrightarrow \frac{\sin x}{\cos x} < \frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} - 1} \quad \text{ـب}$$

$$\Leftrightarrow \frac{(\sqrt{2}-1)\sin x - (\sqrt{3}+\sqrt{2})\cos x}{(\sqrt{2}-1)\cos x} < 0$$

$$\Leftrightarrow \frac{\sqrt{2}(\sin x - \cos x) - (\sin x + \sqrt{3}\cos x)}{(\sqrt{2}-1)\cos x} < 0$$

$$\Leftrightarrow \frac{2\sin\left(x - \frac{\pi}{4}\right) - 2\sin\left(x + \frac{\pi}{3}\right)}{\cos x} < 0$$

$$\Leftrightarrow \frac{\sin\left(x - \frac{\pi}{4}\right) - \sin\left(x + \frac{\pi}{3}\right)}{\cos x} < 0$$

نستعمل الصيغة :  $\sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \left( \frac{p-q}{2} \right)$

$$(E) \Leftrightarrow \frac{2 \cos \left( \frac{x - \frac{\pi}{4} + x + \frac{\pi}{3}}{2} \right) \sin \left( \frac{x - \frac{\pi}{4} - x - \frac{\pi}{3}}{2} \right)}{\cos x} < 0$$

$$\Leftrightarrow \frac{\cos \left( \frac{2x + \frac{\pi}{12}}{2} \right) \sin \left( -\frac{7\pi}{24} \right)}{\cos x} < 0$$

$$\Leftrightarrow \frac{\cos \left( x + \frac{\pi}{24} \right)}{\cos x} > 0$$

لأن  $\sin \left( -\frac{7\pi}{24} \right) < 0$

• نعتبر  $x \in [0, 2\pi]$

$$\cos x > 0 \Leftrightarrow x \in \left[ 0, \frac{\pi}{2} \right[ \cup \left] \frac{3\pi}{2}, 2\pi \right]$$

$$\cos \left( x + \frac{\pi}{24} \right) > 0 \Leftrightarrow x + \frac{\pi}{24} \in \left[ \frac{\pi}{24}, \frac{\pi}{2} \right[ \cup \left] \frac{3\pi}{2}, \frac{49\pi}{24} \right]$$

$$\Leftrightarrow x \in \left[ 0, \frac{11\pi}{24} \right[ \cup \left] \frac{35\pi}{24}, 2\pi \right]$$

• من الجدول التالي :

| X                                                       | 0 | $\frac{11 \pi}{24}$ | $\frac{\pi}{2}$ | $\frac{35 \pi}{24}$ | $\frac{3 \pi}{2}$ | $2 \pi$ |   |
|---------------------------------------------------------|---|---------------------|-----------------|---------------------|-------------------|---------|---|
| COS X                                                   | + | +                   | 0               | —                   | —                 | 0       | + |
| $\cos \left( x + \frac{\pi}{24} \right)$                | + | 0                   | —               | —                   | 0                 | +       | + |
| $\frac{\cos \left( x + \frac{\pi}{24} \right)}{\cos x}$ | + | 0                   | —               | +                   | 0                 | —       | + |

نجد :  $S = \left[0, \frac{11\pi}{24}\right[ \cup \left[\frac{\pi}{2}, \frac{35\pi}{24}\right[ \cup \left[\frac{3\pi}{2}, 2\pi\right]$