

(1) حساب $A(0)$ و $A(\pi)$:

$$A(\pi) = 0 \quad A(0) = -2$$

(2) نستخدم الصيغة : $2 \sin a \cos b = \sin(a+b) + \sin(a-b)$:

$$\begin{aligned} 2 \sin \frac{3x}{2} \cos \frac{x}{2} &= \sin\left(\frac{3x}{2} + \frac{x}{2}\right) + \sin\left(\frac{3x}{2} - \frac{x}{2}\right) \quad \text{نجد:} \\ &= \sin 2x + \sin x \end{aligned}$$

(3) لكل x من $\mathbb{R}$:

$$\begin{aligned} (2 \sin x - 1)(\cos x + 1) &= 2 \sin x \cos x + 2 \sin x - \cos x - 1 \\ &= \sin 2x + \sin x + \sin x - \cos x - 1 \\ &= \sin x + \sin 2x + \sin x - \cos x - 1 \\ &= 2 \sin \frac{3x}{2} \cos \frac{x}{2} + \sin x - \cos x - 1 \end{aligned}$$

$$\begin{aligned} \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) &= \sqrt{2} \left(\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} \right) \quad \text{بما أن:} \\ &= \sqrt{2} \left(\frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x \right) \\ &= \sin x - \cos x \end{aligned}$$

$$\begin{aligned} (2 \sin x - 1)(\cos x + 1) &= 2 \sin \frac{3x}{2} \cos \frac{x}{2} + \sqrt{2} \sin\left(x - \frac{\pi}{4}\right) - 1 \\ &= A(x) \end{aligned}$$

(4) أ- ليكن x من $\mathbb{R}$:

$$A(x) = 0 \Leftrightarrow (2 \sin x - 1)(\cos x + 1) = 0$$

$$\Leftrightarrow 2 \sin x - 1 = 0 \quad \text{أو} \quad \cos x + 1 = 0$$

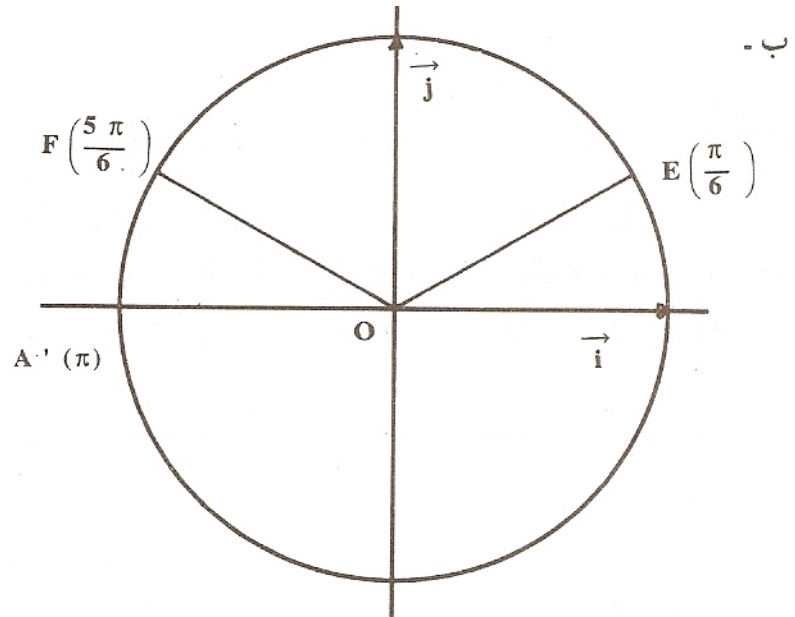
$$2 \sin x - 1 = 0 \Leftrightarrow \sin x = \frac{1}{2} *$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi; k \in \mathbb{Z} \\ x = \frac{5\pi}{6} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

$$\cos x + 1 = 0 \Leftrightarrow \cos x = -1 *$$

$$\Leftrightarrow x = \pi + 2k\pi; k \in \mathbb{Z}$$

$$S = \left\{ \frac{\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\} \cup \{ \pi + 2k\pi / k \in \mathbb{Z} \} *$$



جـ - $A(x) > 0 \Leftrightarrow (2\sin x - 1)(\cos x + 1) > 0$
من الجدول التالي :

x	$-\pi$	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	π
$2\sin x - 1$	—	0	+	—
$\cos x + 1$	0	+	+	0
$A(x)$	0	—	0	—

نجد : $S = \left] \frac{\pi}{6}, \frac{5\pi}{6} \right[$