

(1) لكل $x \in \mathbb{R}$:

$$(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x + 2 \sin x \cos x \\ = 1 + \sin 2x$$

$$\sin 2x + \sin x + \cos x + 1 = 0 \quad (2)$$

$$\Leftrightarrow \sin 2x + 1 + \sin x + \cos x = 0$$

$$\Leftrightarrow (\cos x + \sin x)^2 + (\sin x + \cos x) = 0$$

$$\Leftrightarrow (\cos x + \sin x)(\cos x + \sin x + 1) = 0$$

$$\Leftrightarrow \cos x + \sin x = 0 \text{ أو } \cos x + \sin x + 1 = 0$$

$$\cos x + \sin x = 0 \Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = 0 \quad *$$

$$\Leftrightarrow x - \frac{\pi}{4} = \frac{\pi}{2} + k\pi; k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi; k \in \mathbb{Z}$$

$$\cos x + \sin x + 1 = 0 \Leftrightarrow \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) + 1 = 0 \quad *$$

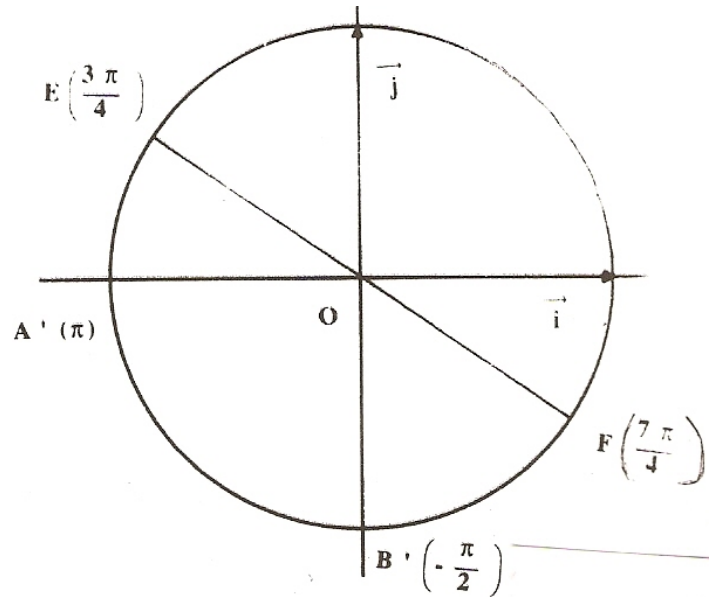
$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \begin{cases} x - \frac{\pi}{4} = \frac{3\pi}{4} + 2k\pi; k \in \mathbb{Z} \\ x - \frac{\pi}{4} = -\frac{3\pi}{4} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \pi + 2k\pi; k \in \mathbb{Z} \\ x = -\frac{\pi}{2} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

$$S = \left\{ \frac{3\pi}{4} + k\pi / k \in \mathbb{Z} \right\} \cup \{ \pi + 2k\pi / k \in \mathbb{Z} \} \cup \left\{ -\frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\} \text{ أخيراً:}$$

• تمثيل الحلول على الدائرة المثلثية:



3) نعتبر المتراجحة : $(E) : x \in [0, \pi]; \sin 2x + \sin x + \cos x + 1 > 0$

$$(E) \Leftrightarrow (\cos x + \sin x) + (\cos x + \sin x + 1) > 0$$

$$\Leftrightarrow \left(\sqrt{2} \cos \left(x - \frac{\pi}{4} \right) \right) \left(\sqrt{2} \cos \left(x - \frac{\pi}{4} \right) + 1 \right) > 0$$

$$\Leftrightarrow \cos \left(x - \frac{\pi}{4} \right) \left(\sqrt{2} \cos \left(x - \frac{\pi}{4} \right) + 1 \right) > 0$$

باستعمال الدائرة المثلثية نجد :

$$\cos \left(x - \frac{\pi}{4} \right) > 0 \Leftrightarrow -\frac{\pi}{4} \leq x - \frac{\pi}{4} \leq \frac{\pi}{2}$$

$$\Leftrightarrow 0 \leq x < \frac{3\pi}{4}$$

$$\cos x + \sin x + 1 > 0 \Leftrightarrow \cos \left(x - \frac{\pi}{4} \right) > -\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow x - \frac{\pi}{4} \in \left[-\frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{4} \right]$$

$$\Leftrightarrow x \in \left[0, \frac{3\pi}{4} \right] \cup \left[\frac{3\pi}{4}, \pi \right]$$

حسب الجدول التالي :

x	0	$\frac{3\pi}{4}$	π
$\cos x + \sin x$	+	0	—
$\cos x + \sin x + 1$	+	0	+
$P(x)$	+	0	—

$S = \left[0, \frac{3\pi}{4} \right]$: نجد