

تمرين 15

$$\begin{aligned}
 & \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{n-1} + \sqrt{n}} \quad (1) \\
 &= \frac{\sqrt{1} - \sqrt{2}}{-1} + \frac{\sqrt{2} - \sqrt{3}}{-1} + \dots + \frac{\sqrt{n-1} - \sqrt{n}}{-1} \\
 &= \frac{1 - \sqrt{n}}{-1} \\
 &= \boxed{\sqrt{n} - 1}
 \end{aligned}$$

(2) بما أن: $\sqrt{n-1} + \sqrt{n} > \sqrt{n}$

فإن: $\frac{1}{\sqrt{n-1} + \sqrt{n}} < \frac{1}{\sqrt{n}}$

إذن: $\frac{1}{\sqrt{2} + \sqrt{3}} < \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{1} + \sqrt{2}} < \frac{1}{\sqrt{1}}$

بجمع طرفي التفاوتات طرفا بطرف نحصل على:

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{n-1} + \sqrt{n}} < \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}$$

وحسب السؤال السابق:

$$\sqrt{n} - 1 < \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}$$

ومنه فإن: $\boxed{\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}}$