

1- التحقق من المتساوية المقترحة

ليكن t عنصرا من $\mathbb{R} - \left\{-2, \frac{1}{2}\right\}$

$$\frac{2}{t+2} + \frac{1}{2t-1} = \frac{2(2t-1)+t+2}{(t+2)(2t-1)} \quad \text{لدينا}$$

$$= \frac{5t}{2t^2+3t-2}$$

$$\frac{t}{2t^2+3t-2} = \frac{1}{5} \left(\frac{2}{t+2} + \frac{1}{2t-1} \right) \quad \text{ومنه}$$

2- حساب التكامل I

$$I = \int_1^4 \frac{1}{5} \left(\frac{2}{t+2} + \frac{1}{2t-1} \right) dt \quad \text{لدينا}$$

$$= \frac{2}{5} \int_1^4 \frac{1}{t+2} dt + \frac{1}{10} \int_1^4 \frac{2}{2t-1} dt$$

$$= \frac{2}{5} [\ln(t+2)]_1^4 + \frac{1}{10} [\ln(2t-1)]_1^4$$

$$= \frac{2}{5} (\ln 6 - \ln 3) + \frac{1}{10} (\ln 7 - \ln 1)$$

$$= \frac{2}{5} \ln 2 + \frac{1}{10} \ln 7$$

3- الاستنتاج

نضع $t = \sqrt{3x+1}$ ويكون لدينا $x = \frac{1}{3}(t^2-1)$

$$dx = \frac{2}{3} t dt \quad \text{أي} \quad dt = \frac{3}{2\sqrt{3x+1}} dx$$

إذا كان $x=0$ فإن $t=1$

إذا كان $x=5$ فإن $t=4$

$$\int_0^5 \frac{dx}{2x + \sqrt{3x+1}} = \int_1^4 \frac{1}{\frac{2}{3}(t^2-1)+t} \cdot \frac{2}{3} t dt \quad \text{ومنه}$$

$$= 2 \int_1^4 \frac{t}{2t^2+3t-2} dt$$

$$= 2I$$

$$= \frac{4}{5} \ln 2 + \frac{1}{5} \ln 7$$