

نعلم أنه إذا كان $z_1 + z_2 = s$ و $z_1 \cdot z_2 = p$

فإن z_1 و z_2 هما حللي المعادلة $Z^2 - sZ + p = 0$

(1) لنحدد z_1 و z_2 حيث : $z_1 + z_2 = 2\sqrt{3}$ و $z_1 \cdot z_2 = -4$

لدينا z_1 و z_2 هما حللي المعادلة : $z^2 - 2\sqrt{3}z - 4 = 0$

$$\Delta = 28 = (2\sqrt{7})^2$$

إذن $(z_2 = \sqrt{3} - \sqrt{7} \text{ و } z_1 = \sqrt{3} + \sqrt{7})$

أو $(z_1 = \sqrt{3} + \sqrt{7} \text{ و } z_2 = \sqrt{3} - \sqrt{7})$

إذن

$$(z_1, z_2) \in \{(\sqrt{3} + \sqrt{7}, \sqrt{3} - \sqrt{7}); (\sqrt{3} - \sqrt{7}, \sqrt{3} + \sqrt{7})\}$$

(2) لنحدد z_1 و z_2 حيث : $z_1 - z_2 = -2 + 2\sqrt{2}$ و $z_1 \cdot z_2 = -6$

لدينا :

$$\begin{cases} z_1 - z_2 = -2 + 2\sqrt{2} \\ z_1 \cdot z_2 = -6 \end{cases} \Leftrightarrow \begin{cases} z_1 + (-z_2) = -2 + 2\sqrt{2} \\ z_1 \cdot (-z_2) = 6 \end{cases}$$

إذن z_1 و $(-z_2)$ هما حللي المعادلة $z^2 - (2 - 2\sqrt{2})z + 6 = 0$

$$\begin{aligned}\Delta &= (2 - 2\sqrt{2})^2 - 24 \\ &= -12 - 8\sqrt{2} \\ &= -4(3 + 2\sqrt{2}) \\ &= [2i(1 + \sqrt{2})]^2\end{aligned}$$

إذن
 $(-z_2 = (1 - \sqrt{2}) - (1 + \sqrt{2})i \text{ و } z_1 = (1 - \sqrt{2}) + (1 + \sqrt{2})i)$

أو
 $(-z_2 = (1 - \sqrt{2}) + (1 + \sqrt{2})i \text{ و } z_1 = (1 - \sqrt{2}) - (1 + \sqrt{2})i)$

أي
 $(z_2 = (\sqrt{2} - 1) + (1 + \sqrt{2})i \text{ و } z_1 = (1 - \sqrt{2}) + (1 + \sqrt{2})i)$

أو
 $(z_2 = (\sqrt{2} - 1) - (1 + \sqrt{2})i \text{ و } z_1 = (1 - \sqrt{2}) - (1 + \sqrt{2})i)$

إذن

$$(z_1, z_2) \in \left\{ \left((1 - \sqrt{2}) + (1 + \sqrt{2})i, (\sqrt{2} - 1) + (1 + \sqrt{2})i \right); \right. \\ \left. \left((1 - \sqrt{2}) - (1 + \sqrt{2})i, (\sqrt{2} - 1) - (1 + \sqrt{2})i \right) \right\}$$

$$\begin{cases} \frac{1}{z_1} + \frac{1}{z_2} = \frac{4}{5} \\ z_1 \cdot z_2 = 5 \end{cases} \Leftrightarrow \begin{cases} \frac{z_1 + z_2}{z_1 \cdot z_2} = \frac{4}{5} \\ z_1 \cdot z_2 = 5 \end{cases} \quad (3)$$

$$\Leftrightarrow \begin{cases} z_1 + z_2 = 4 \\ z_1 \cdot z_2 = 5 \end{cases}$$

إذن z_1 و z_2 هما حللي المعادلة $z^2 - 4z + 5 = 0$

$$\Delta = -4 = (2i)^2$$

إذن

$$(z_1, z_2) \in \{(2 + i, 2 - i); (2 - i, 2 + i)\}$$

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