

### 1- حساب القيمتين

ليكن  $x$  عنصرا من  $\mathbb{R}$

$$u'(x) = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{x^2+1}}$$

$$v'(x) = -\frac{u'(x)}{u^2(x)} = -\frac{x}{(x^2+1)\sqrt{x^2+1}}$$

### 2- حساب التكامل $I(\alpha)$

$$I(\alpha) = \int_0^\alpha \frac{x}{(1+x^2)\sqrt{1+x^2}} x^2 dx \quad \text{لدينا}$$

$$= \int_0^\alpha -v'(x) x^2 dx$$

$$= -([v(x) x^2]_0^\alpha + \int_0^\alpha v(x) (2x) dx)$$

$$= -\alpha^2 V(\alpha) + 2 \int_0^\alpha \frac{x}{\sqrt{1+x^2}} dx$$

$$= -\alpha^2 V(\alpha) + 2 \int_0^\alpha u'(x) dx$$

$$= -\alpha^2 V(\alpha) + 2[u(x)]_0^\alpha$$

$$= -\alpha^2 V(\alpha) + 2(u(\alpha) - u(0))$$

$$I(\alpha) = \frac{-\alpha^2}{\sqrt{1+\alpha^2}} + 2\sqrt{1+\alpha^2} - 2 \quad \text{إذن}$$

### 3- حساب التكامل $J$

نضع  $x = \ln t$  ويكون لدينا  $dx = \frac{1}{t} dt$  أي  $dt = e^x dx$

إذا كان  $t = e$  فإن  $x = 1$  وإذا كان  $t = 1$  فإن  $x = 0$  ومنه

$$J = \int_0^1 \frac{x^3}{e^x (1+x^2) \sqrt{1+x^2}} e^x dx = \int_0^1 \frac{x^3}{(1+x^2) \sqrt{1+x^2}} dx$$

$$J = I(1) = \frac{3\sqrt{2}-4}{2} \quad \text{إذن}$$