

$$I = \int_0^1 \left(\frac{1}{(x+1)^2} + \frac{1}{2x+1} \right) dx \quad \text{* حساب}$$

$$I = \int_0^1 \left(\frac{(x+1)'}{(x+1)^2} + \frac{1}{2} \cdot \frac{(2x+1)'}{2x+1} \right) dx \quad \text{لدينا :}$$

$$= \left[\frac{-1}{x+1} + \frac{1}{2} \ln(2x+1) \right]_0^1 = -\frac{1}{2} + \frac{1}{2} \ln 3 + 1$$

$$I = \frac{1}{2} (1 + \ln 3) \quad \text{إذن :}$$

$$J = \int_0^1 \frac{x+1}{\sqrt{x+1}} dx \quad \text{* حساب}$$

$$\sqrt{x} = t - 1 \quad \text{أي :} \quad t = \sqrt{x+1} \quad \text{نضع :}$$

$$x = (t-1)^2 \quad \text{ومنه :}$$

$$dx = 2(t-1) dt \quad \text{وبالتالي :}$$

$$t = 1 \quad \text{لدينا :} \quad x = 0 \quad \text{من أجل}$$

$$t = 2 \quad \text{لدينا :} \quad x = 1 \quad \text{ومن أجل}$$

$$J = \int_1^2 \frac{(t-1)^2 + 1}{t} 2(t-1) dt \quad \text{إذن :}$$

$$= 2 \int_1^2 \frac{(t^2 - 2t + 2)}{t} (t-1) dt = 2 \int_1^2 \frac{t^3 - t^2 - 2t^2 + 2t + 2t - 2}{t} dt$$

$$= 2 \int_1^2 \frac{t^3 - 3t^2 + 4t - 2}{t} dt = 2 \int_1^2 \left(t^2 - 3t + 4 - \frac{2}{t} \right) dt$$

$$= 2 \left[\frac{t^3}{3} - \frac{3t^2}{2} + 4t - 2 \ln t \right]_1^2$$

$$= 2 \left(\frac{8}{3} - 6 + 8 - 2 \ln 2 - \frac{1}{3} + \frac{3}{2} - 4 \right)$$

$$J = \frac{11}{3} - 4 \ln 2$$

وبالتالي فإن :

$$K = \int_0^1 3x^2 \ln(x^2 + 1) dx$$

* حساب

نضع :

$$V' = 3x^2 \quad \text{و} \quad U = \ln(x^2 + 1)$$

$$V = x^3 \quad \text{' } \quad U' = \frac{2x}{x^2 + 1}$$

إذن :

$$K = [x^3 \ln(x^2 + 1)]_0^1 - 2 \int_0^1 \frac{x^4}{x^2 + 1} dx$$

$$= \ln 2 - 2 \int_0^1 \left(x^2 - 1 + \frac{1}{x^2 + 1} \right) dx$$

$$= \ln 2 - 2 \left[\frac{x^3}{3} - x + \text{Arc tan } x \right]_0^1$$

$$= \ln 2 - 2 \left(\frac{1}{3} - 1 + \frac{\pi}{4} \right)$$

وبالتالي فإن :

$$K = \frac{4}{3} - \frac{\pi}{2} + \ln 2$$

Achamel



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