

$$\begin{cases} u_0 = 1 \\ u_{n+1} = \frac{2u_n + 3}{2u_n + 7} ; n \in \mathbb{N} \end{cases}$$

(1) أ. • لدينا $u_0 = 1$ إذن $\frac{1}{2} < u_0 \leq 1$

• نفترض أن $\frac{1}{2} < u_p \leq 1$ لنبين أن $\frac{1}{2} < u_{p+1} \leq 1$ لدينا :

$$u_{p+1} = \frac{2u_p + 3}{2u_p + 7} = 1 - \frac{4}{2u_p + 7}$$

$$\frac{1}{2} < u_p \leq 1 \Rightarrow 8 < 2u_p + 7 \leq 9$$

$$\Rightarrow -\frac{1}{2} < -\frac{4}{2u_p + 7} \leq -\frac{4}{9}$$

$$\Rightarrow \frac{1}{2} < 1 - \frac{4}{2u_p + 7} \leq \frac{5}{9}$$

$$\Rightarrow \frac{1}{2} < u_{p+1} \leq 1$$

$$\boxed{\forall n \in \mathbb{N} ; \frac{1}{2} < u_n \leq 1}$$

إذن :

$$u_{n+1} - u_n = \frac{2u_n + 3}{2u_n + 7} - u_n$$

ب. *

$$= \frac{-2u_n^2 - 5u_n + 3}{2u_n + 7}$$

$$= \frac{(1 - 2u_n)(u_n + 3)}{2u_n + 7}$$

$$\frac{1}{2} < u_n \leq 1 \Rightarrow \begin{cases} 1 - 2u_n < 0 \\ u_n + 3 > 0 \\ 2u_n + 7 > 0 \end{cases}$$

$$\Rightarrow \frac{(1 - 2u_n)(3 + u_n)}{2u_n + 7} < 0$$

$$\forall n \in \mathbb{N} \quad ; \quad u_{n+1} < u_n$$

إذن

ومنه فإن المتتالية (u_n) تناقصية.

* بما أن المتتالية (u_n) تناقصية ومصغرة بالعدد $\frac{1}{2}$ فإنها متقاربة.

$$(2) \bullet \text{ لدينا : } u_{n+1} - \frac{1}{2} = \frac{2u_n + 3}{2u_n + 7} - \frac{1}{2} = \frac{2u_n - 1}{2(2u_n + 7)}$$

$$u_{n+1} - \frac{1}{2} = \frac{1}{2u_n + 7} \left(u_n - \frac{1}{2} \right)$$

إذن :

$$\frac{1}{2} < u_n \Rightarrow 8 \leq 2u_n + 7$$

$$\Rightarrow \frac{1}{2u_n + 7} \leq \frac{1}{8}$$

$$\Rightarrow \frac{u_n - \frac{1}{2}}{2u_n + 7} \leq \frac{1}{8} \left(u_n - \frac{1}{2} \right)$$

$$\Rightarrow u_{n+1} - \frac{1}{2} \leq \frac{1}{8} \left(u_n - \frac{1}{2} \right)$$

$$\forall n \in \mathbb{N} \quad ; \quad u_{n+1} - \frac{1}{2} \leq \frac{1}{8} \left(u_n - \frac{1}{2} \right) \quad \text{إذن :}$$

$$0 < u_1 - \frac{1}{2} \leq \frac{1}{8} \left(u_0 - \frac{1}{2} \right) \quad \text{ب. - *}$$

$$\times \quad 0 < u_2 - \frac{1}{2} \leq \frac{1}{8} \left(u_1 - \frac{1}{2} \right)$$

⋮

$$\times \quad 0 < u_n - \frac{1}{2} \leq \frac{1}{8} \left(u_{n-1} - \frac{1}{2} \right)$$

$$\hline 0 < u_n - \frac{1}{2} \leq \left(\frac{1}{8} \right)^n \left(u_0 - \frac{1}{2} \right)$$

$$u_0 - \frac{1}{2} = \frac{1}{2} \quad , \quad \frac{1}{8} = \left(\frac{1}{2}\right)^3 \quad \text{وبما أن}$$

$$\left| u_n - \frac{1}{2} \right| \leq \left(\frac{1}{2}\right)^{3n} \cdot \frac{1}{2} \quad \text{فإن}$$

$$\boxed{\forall n \in \mathbb{N} \quad ; \quad \left| u_n - \frac{1}{2} \right| \leq \left(\frac{1}{2}\right)^{3n+1}} \quad \text{أي}$$

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{2}\right)^{3n+1} = 0 \quad \text{فإن} \quad -1 < \frac{1}{2} < 1 \quad * \text{ بما أن}$$

$$\boxed{\lim_{n \rightarrow +\infty} u_n = \frac{1}{2}}$$

وبالتالي فإن