

$$\begin{aligned}
 z &= \frac{1 + e^{i\theta}}{1 - e^{i\theta}} ; \quad \theta \in]0, \pi[\\
 z &= \frac{1 + e^{i\theta}}{1 - e^{i\theta}} = \frac{1 + \cos \theta + i \sin \theta}{1 - \cos \theta - i \sin \theta} \quad (1) \\
 &= \frac{2 \cos^2 \frac{\theta}{2} + 2 i \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2} - 2 i \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \\
 &= \frac{2 \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right)}{2 \sin \frac{\theta}{2} \left(\sin \frac{\theta}{2} - i \cos \frac{\theta}{2} \right)} \\
 &= \frac{1}{\tan \frac{\theta}{2}} \cdot \frac{\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}}{-i^2 \sin \frac{\theta}{2} - i \cos \frac{\theta}{2}} \\
 &= \frac{1}{\tan \frac{\theta}{2}} \cdot \frac{\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}}{-i \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right)} \\
 &= \frac{-i^2}{-i \tan \frac{\theta}{2}}
 \end{aligned}$$

$$z = \frac{i}{\tan \frac{\theta}{2}}$$

إذن

$$|z| = \left| \frac{i}{\tan \frac{\theta}{2}} \right| = \left| \frac{1}{\tan \frac{\theta}{2}} \right| \quad (2) \text{ لدينا}$$

بما أن $0 < \theta < \pi$ فإن $0 < \frac{\theta}{2} < \frac{\pi}{2}$ ومنه فإن $\tan \frac{\theta}{2} > 0$

$$|z| = \frac{1}{\tan \frac{\theta}{2}}$$

إذن

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