

$$\begin{cases} u_1 = \frac{3}{2} \\ u_{n+1} = \frac{1}{2} u_n + \frac{n-1}{2n(n+1)} \end{cases} ; \quad n \in \mathbb{N}^*$$

(1) لدينا $u_3 = \frac{11}{24}$ و $u_2 = \frac{3}{4}$
إذن $u_1 > u_2 > u_3$

$$\begin{aligned} u_{n+1} - u_n &= \frac{1}{2} u_n + \frac{n-1}{2n(n+1)} - \frac{1}{2} u_{n-1} - \frac{n-2}{2(n-1)n} \quad - a \\ &= \frac{1}{2} (u_n - u_{n-1}) + \frac{(n-1)^2 - (n+1)(n-2)}{2n(n-1)(n+1)} \\ &= \frac{1}{2} (u_n - u_{n-1}) + \frac{n^2 - 2n + 1 - n^2 + 2n - n + 2}{2n(n-1)(n+1)} \end{aligned}$$

$$u_{n+1} - u_n = \frac{1}{2} (u_n - u_{n-1}) + \frac{3-n}{2n(n-1)(n+1)} \quad \text{إذن}$$

- b لدينا $u_1 > u_2 > u_3$
نفترض أن $u_p - u_{p-1} \leq 0$ مع $p \geq 3$
لنبين أن $u_{p+1} - u_p \leq 0$

$$\begin{aligned} \begin{cases} u_p - u_{p-1} \leq 0 \\ p \geq 3 \end{cases} &\Rightarrow \begin{cases} \frac{1}{2} (u_p - u_{p-1}) \leq 0 \\ \frac{3-p}{2p(p-1)(p+1)} \leq 0 \end{cases} \\ &\Rightarrow \frac{1}{2} (u_p - u_{p-1}) + \frac{3-p}{2p(p-1)(p+1)} \leq 0 \\ &\Rightarrow u_{p+1} - u_p \leq 0 \end{aligned}$$

$$\forall n \in \mathbb{N}^* ; \quad u_{n+1} \leq u_n \quad \text{إذن}$$

ومنه فإن المتتالية $(u_n)_{n \geq 1}$ تناقصية.

$$V_n = u_n - \frac{1}{n} ; \quad n \in \mathbb{N}^* \quad (3)$$

$$\begin{aligned} V_{n+1} &= u_{n+1} - \frac{1}{n+1} \quad - a \\ &= \frac{1}{2} u_n + \frac{n-1}{2n(n+1)} - \frac{1}{n+1} \\ &= \frac{1}{2} u_n + \frac{n-1-2n}{2n(n+1)} \\ &= \frac{1}{2} \left(u_n - \frac{1}{n} \right) \end{aligned}$$

$$\boxed{\forall n \in \mathbb{N}^* ; V_{n+1} = \frac{1}{2} V_n} \quad \text{إذن}$$

ومنه فإن $(V_n)_{n \geq 1}$ متتالية هندسية أساسها $\frac{1}{2}$.

- b بما أن $(V_n)_{n \geq 1}$ متتالية هندسية أساسها $q = \frac{1}{2}$ وحدها الأول $V_1 = \frac{1}{2}$.

$$\boxed{\forall n \in \mathbb{N}^* ; V_n = \left(\frac{1}{2}\right)^n} \quad \text{فإن}$$

$$V_n = u_n - \frac{1}{n} \Rightarrow u_n = V_n + \frac{1}{n}$$

$$\boxed{\forall n \in \mathbb{N}^* ; u_n = \frac{1}{n} + \left(\frac{1}{2}\right)^n} \quad \text{إذن}$$

$$\left. \begin{array}{l} \lim_{n \rightarrow +\infty} \frac{1}{n} = 0 \\ \lim_{n \rightarrow +\infty} \left(\frac{1}{2}\right)^n = 0 \end{array} \right\} \Rightarrow \boxed{\lim_{n \rightarrow +\infty} u_n = 0} \quad - c$$