

$$I = \int_0^1 \frac{2x}{(x+1)^2 (x^2+1)} dx \quad \text{1- حساب}$$

$$I = \int_0^1 \frac{(x+1)^2 - (x^2+1)}{(x+1)^2 (x^2+1)} dx \quad \text{لدينا :}$$

$$= \int_0^1 \left(\frac{1}{x^2+1} - \frac{1}{(x+1)^2} \right) dx = \left[\text{Arctan } x + \frac{1}{x+1} \right]_0^1$$

$$= \frac{\pi}{4} + \frac{1}{2} - 0 - 1 = \frac{\pi - 2}{4}$$

$$J = \int_0^{\frac{\pi}{4}} \frac{x}{\cos^2 x} dx \quad \text{2- حساب}$$

$$V' = \frac{1}{\cos^2 x} \quad \text{و} \quad U = x \quad \text{نضع}$$

$$V = \tan x \quad , \quad U' = 1$$

$$J = [x \tan x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x dx \quad \text{إذن :}$$

$$= \frac{\pi}{4} - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx = \frac{\pi}{4} + \int_0^{\frac{\pi}{4}} \frac{(\cos x)'}{\cos x} dx$$

$$= \frac{\pi}{4} + [\ln |\cos x|]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2} = \frac{\pi}{4} + \ln \sqrt{\frac{1}{2}} = \frac{\pi}{4} + \frac{1}{2} \ln \frac{1}{2}$$

$$J = \frac{\pi}{4} - \frac{1}{2} \ln 2 \quad \text{وبالتالي فإن :}$$

$$K = \int_0^{\ln 2} e^x \ln (e^x + \sqrt{e^{2x} + 1}) dx \quad \text{3- حساب}$$

$$V' = e^x \quad \text{و} \quad U = \ln (e^x + \sqrt{e^{2x} + 1}) \quad \text{نضع :}$$

$$V = e^x \quad , \quad U' = \frac{e^x + \frac{e^{2x}}{\sqrt{e^{2x} + 1}}}{e^x + \sqrt{e^{2x} + 1}} = \frac{e^x}{\sqrt{e^{2x} + 1}}$$

$$K = [e^x \ln (e^x + \sqrt{e^{2x} + 1})]_0^{\ln 2} - \int_0^{\ln 2} \frac{e^{2x}}{\sqrt{e^{2x} + 1}} dx \quad \text{إذن :}$$

$$= 2 \ln (2 + \sqrt{5}) - \ln (1 + \sqrt{2}) - [\sqrt{e^{2x} + 1}]_0^{\ln 2}$$

$$= 2 \ln (2 + \sqrt{5}) - \ln (1 + \sqrt{2}) - (\sqrt{5} - \sqrt{2})$$

$$K = \sqrt{2} - \sqrt{5} + 2 \ln (2 + \sqrt{5}) - \ln (1 + \sqrt{2}) \quad \text{وبالتالي فإن :}$$