

$$\begin{aligned} 1 - \text{أ- لدينا} : \frac{2(\cos \theta - 1) + \frac{1}{1 + \cos \theta}}{1 + \cos \theta} &= \frac{2(\cos \theta - 1)(\cos \theta + 1) + 1}{1 + \cos \theta} \\ &= \frac{2(\cos^2 \theta - 1) + 1}{1 + \cos \theta} = \frac{2\cos^2 \theta - 1}{1 + \cos \theta} = \frac{\cos 2\theta}{1 + \cos \theta} \end{aligned}$$

$$\text{ب- نعلم أن : } 1 + \cos 2\alpha = 2\cos^2 \alpha$$

$$\begin{aligned} 1 + \cos \theta &= 1 + \cos \left(2 \cdot \frac{\theta}{2} \right) \quad \text{إذن :} \\ &= 2\cos^2 \frac{\theta}{2} \end{aligned}$$

$$\text{2- نضع : } \theta = x - \frac{\pi}{4} \quad \text{أي : } x = \theta + \frac{\pi}{4}$$

$$\text{ومنه : } dx = d\theta$$

$$\text{من أجل } x = 0 \quad \text{لدينا : } \theta = -\frac{\pi}{4}$$

$$\text{ومن أجل } x = \frac{\pi}{2} \quad \text{لدينا : } \theta = \frac{\pi}{4}$$

إذن :

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin \left[2 \left(\theta + \frac{\pi}{4} \right) \right]}{1 + \cos \theta} d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin \left(2\theta + \frac{\pi}{2} \right) d\theta}{1 + \cos \theta} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2\theta}{1 + \cos \theta} d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[2(\cos \theta - 1) + \frac{1}{1 + \cos \theta} \right] d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(2(\cos \theta - 1) + \frac{1}{2 \cos^2 \frac{\theta}{2}} \right) d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(2 \cos \theta - 2 + \frac{\left(\frac{\theta}{2} \right)'}{\cos^2 \frac{\theta}{2}} \right) d\theta = \left[2 \sin \theta - 2\theta + \tan \frac{\theta}{2} \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= 2 \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{\pi}{4} + \tan \frac{\pi}{8} + 2 \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{\pi}{4} + \tan \frac{\pi}{8}$$

$$= 2\sqrt{2} - \pi + 2 \tan \frac{\pi}{8}$$

بقي أن نحسب : $\tan \frac{\pi}{8}$

نعلم أن :

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

إذن :

$$\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} = \tan \frac{\pi}{4}$$

أي : $\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} = 1$ أي : $\tan^2 \frac{\pi}{8} + 2 \tan \frac{\pi}{8} - 1 = 0$

أي : $\left(\tan \frac{\pi}{8} + 1 \right)^2 = 2$ أي : $\tan \frac{\pi}{8} + 1 = \sqrt{2}$ (لأن $\tan \frac{\pi}{8} + 1 > 0$)

ومنه : $\tan \frac{\pi}{8} = \sqrt{2} - 1$

وبالتالي فإن :

$$I = 2\sqrt{2} - \pi + 2(\sqrt{2} - 1)$$

$$= 4\sqrt{2} - 2 - \pi$$