

$$(E) : (4 \cos \theta) z^2 - (2 \cos (2\theta)) z + i \sin \theta = 0$$

$$z_1 = \frac{1}{2} (\cos \theta - i \sin \theta) \quad * (1)$$

لدينا :

$$\begin{aligned} & 4 \cos \theta \left[\frac{1}{2} (\cos \theta - i \sin \theta) \right]^2 - (2 \cos (2\theta)) \left[\frac{1}{2} (\cos \theta - i \sin \theta) \right] + i \sin \theta \\ &= \cos \theta (\cos 2\theta - i \sin 2\theta) - (\cos 2\theta) (\cos \theta - i \sin \theta) + i \sin \theta \\ &= \cos \theta \cos 2\theta - i \cos \theta \sin 2\theta - \cos 2\theta \cos \theta + i \sin \theta \cos 2\theta + i \sin \theta \\ &= i (\sin \theta \cos 2\theta - \cos \theta \sin 2\theta) + i \sin \theta \\ &= i (\sin (\theta - 2\theta)) + i \sin \theta \\ &= -i \sin \theta + i \sin \theta \\ &= 0 \end{aligned}$$

إذن z_1 حل للمعادل (E).

$$z_2 = \frac{1}{2} \tan \theta (-\sin \theta + i \cos \theta) \quad *$$

$$= \frac{1}{2} \tan \theta (\cos \theta + i \sin \theta)$$

$$4 \cos \theta \left[\frac{1}{2} i \tan \theta (\cos \theta + i \sin \theta) \right]^2 - 2 \cos 2\theta \quad \text{لدينا :}$$

$$\left[\frac{1}{2} i \tan \theta (\cos \theta + i \sin \theta) + i \sin \theta \right]$$

$$\begin{aligned}
 &= -\cos \theta \cdot \tan^2 \theta (\cos 2\theta + i \sin 2\theta) \\
 &\quad - \cos(2\theta) \cdot i \cdot \tan \theta (\cos \theta + i \sin \theta) + i \sin \theta \\
 &= -\cos \theta \cdot \tan^2 \theta \cdot \cos(2\theta) - i \cos \theta \cdot \tan^2 \theta \cdot \sin(2\theta) \\
 &\quad - i \cos(2\theta) \cdot \cos \theta \tan \theta + \cos(2\theta) \cdot \tan \theta \cdot \sin \theta + i \sin \theta \\
 &= \cos(2\theta) \cdot \tan \theta (\sin \theta - \cos \theta \cdot \tan \theta) \\
 &\quad + i \left[\sin \theta - \cos \theta \cdot \tan^2 \theta \cdot \sin 2\theta - \cos 2\theta \cdot \cos \theta \cdot \tan \theta \right] \\
 &= \cos(2\theta) \cdot \tan \theta \left(\sin \theta - \cos \theta \cdot \frac{\sin \theta}{\cos \theta} \right) \\
 &\quad + i \left[\sin \theta - \cos \theta \cdot \tan \theta (\tan \theta \cdot \sin 2\theta + \cos 2\theta) \right] \\
 &= \cos(2\theta) \cdot \tan \theta (\sin \theta - \sin \theta) \\
 &\quad + i \left[\sin \theta - \sin \theta (\tan \theta \cdot 2 \sin \theta \cdot \cos \theta + 2 \cos^2 \theta - 1) \right] \\
 &= i \left[\sin \theta - \sin \theta (2 \sin^2 \theta + 2 \cos^2 \theta - 1) \right] \\
 &= i \left[\sin \theta - \sin \theta \cdot 2(\cos^2 \theta + \sin^2 \theta) - 1 \right] \\
 &= i (\sin \theta - \sin \theta) (2 - 1) \\
 &= 0
 \end{aligned}$$

إذن z_2 حل للمعادلة (E).

$$\begin{aligned}
 z_1 &= \frac{1}{2} (\cos \theta - i \sin \theta) \quad * (2) \\
 &= \frac{1}{2} (\cos(-\theta) + i \sin(-\theta))
 \end{aligned}$$

$$\boxed{z_1 = \left[\frac{1}{2}, -\theta \right] = \frac{1}{2} e^{-i\theta}} \quad \text{إذن}$$

$$\begin{aligned}
 z_2 &= \frac{1}{2} \tan \theta (-\sin \theta + i \cos \theta) \\
 &= \frac{1}{2} i \tan \theta (\cos \theta + i \sin \theta) \\
 &= \left(\frac{1}{2} \tan \theta \right) e^{i \frac{\pi}{2}} \cdot e^{i\theta} \\
 &= \frac{1}{2} \tan \theta \cdot e^{i\left(\theta + \frac{\pi}{2}\right)}
 \end{aligned}$$

بما أن $\tan \theta > 0$ فإن $0 < \theta < \frac{\pi}{2}$

$$\boxed{z_2 = \left[\frac{1}{2} \tan \theta ; \frac{\pi}{2} + \theta \right] = \frac{1}{2} \tan \theta \cdot e^{i\left(\frac{\pi}{2} + \theta\right)}} \quad \text{ومنه فإن}$$

$$\begin{aligned}
 (O, A, B \text{ مستقيمة}) &\Leftrightarrow \frac{z_2 - 0}{z_1 - 0} \in \mathbb{R} \\
 &\Leftrightarrow \frac{\frac{1}{2} \tan \theta \cdot e^{i\left(\frac{\pi}{2} + \theta\right)}}{\frac{1}{2} \cdot e^{-i\theta}} \in \mathbb{R} \\
 &\Leftrightarrow \tan \theta \cdot e^{i\left(\frac{\pi}{2} + 2\theta\right)} \in \mathbb{R} \\
 &\Leftrightarrow \frac{\pi}{2} + 2\theta = k\pi ; k \in \mathbb{Z}
 \end{aligned}$$

$$\Leftrightarrow 2\theta = -\frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow \theta = -\frac{\pi}{4} + k\frac{\pi}{2}; k \in \mathbb{Z}$$

$$0 < \theta < \frac{\pi}{2} \quad \text{بما أن :}$$

$$0 < -\frac{\pi}{4} + k\frac{\pi}{2} < \frac{\pi}{2} \quad \text{فإن :}$$

$$\frac{\pi}{4} < k\frac{\pi}{2} < \frac{\pi}{2} + \frac{\pi}{4} \quad \text{أي :}$$

$$\frac{1}{4} < \frac{k}{2} < \frac{1}{2} + \frac{1}{4} \quad \text{ومنه فإن :}$$

$$\frac{1}{2} < k < \frac{3}{2} \quad \text{إذن :}$$

$$k = 1$$

إذن :

$$\theta = \frac{\pi}{4} \quad \text{إذن : O و A و B مستقيمة إذا كان}$$

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