

$$(E) : z^2 - \left(4 \cos^2 \frac{\theta}{2} \right) z + 4 \cos^4 \frac{\theta}{2} + \sin^2 \theta = 0$$

(1) لدينا :

$$\begin{aligned} 4 \cos^4 \frac{\theta}{2} + \sin^2 \theta &= 4 \cos^4 \frac{\theta}{2} + \left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right)^2 \\ &= 4 \cos^2 \frac{\theta}{2} \left(\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right) \\ &= 4 \cos^2 \frac{\theta}{2} \end{aligned}$$

إذن المعادلة (E) تصبح :

$$z^2 - \left(4 \cos^2 \frac{\theta}{2} \right) z + 4 \cos^2 \frac{\theta}{2} = 0$$

$$\begin{aligned} \Delta &= \left(4 \cos^2 \frac{\theta}{2} \right)^2 - 4 \cdot \cos^2 \frac{\theta}{2} \\ &= 16 \cos^2 \frac{\theta}{2} \left(\cos^2 \frac{\theta}{2} - 1 \right) \\ &= -16 \cos^2 \frac{\theta}{2} \sin^2 \frac{\theta}{2} \\ &= \left(4 i \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right)^2 \end{aligned}$$

$$z_1 = \frac{4 \cos^2 \frac{\theta}{2} + 4 i \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2}$$

$$z_1 = 2 \cos^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$z_2 = 2 \cos^2 \frac{\theta}{2} - 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

(2) نلاحظ أن $z_2 = \overline{z_1}$

$$\begin{aligned} z_1 &= 2 \cos^2 \frac{\theta}{2} + 2 i \sin \frac{\theta}{2} \cos \frac{\theta}{2} \\ &= 2 \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right) \end{aligned}$$

بما أن $-\pi < \theta < \pi$ فإن $-\frac{\pi}{2} < \frac{\theta}{2} < \frac{\pi}{2}$ فإن $\cos \frac{\theta}{2} > 0$

ومنه فإن

$$\begin{aligned} |z_1| &= |z_2| = 2 \cos \frac{\theta}{2} \\ \text{Arg } z_1 &\equiv \frac{\theta}{2} \quad [2\pi] \quad \text{و} \\ \text{Arg } z_2 &\equiv -\frac{\theta}{2} \quad [2\pi] \quad \text{و} \end{aligned}$$

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