

(1) لكل $x \in \mathbb{R}$ لدينا:

$$\begin{aligned}
 & 2(\cos x - \sin x) \left(\cos x - \frac{\sqrt{3}}{2} \right) \\
 &= 2 \left(\cos^2 x - \frac{\sqrt{3}}{2} \cos x - \sin x \cos x + \frac{\sqrt{3}}{2} \sin x \right) \\
 &= 2 \cos^2 x - \sqrt{3} \cos x - 2 \sin x \cos x + \sqrt{3} \sin x \\
 &= 2 \cos^2 x - 2 \sin x \cos x - \sqrt{3} \cos x + \sqrt{3} \sin x \\
 &= 1 + \cos 2x - \sin 2x - \sqrt{3} \cos x + \sqrt{3} \sin x \\
 &= \cos 2x - \sin 2x - \sqrt{3} \cos x + \sqrt{3} \sin x + 1 \\
 &= \cos 2x - \sin 2x - \sqrt{3} (\cos x - \sin x) + 1 \\
 &= \cos 2x - \sin 2x - \sqrt{3} \left(\sqrt{2} \cos \left(x + \frac{\pi}{4} \right) \right) + 1 \\
 &= \cos 2x - \sin 2x - \sqrt{6} \cos \left(x + \frac{\pi}{4} \right) + 1 \\
 &= A(x)
 \end{aligned}$$

(2) $A(x) = 0 \Leftrightarrow \cos x - \sin x = 0$ أو $\cos x - \frac{\sqrt{3}}{2} = 0$

$$\cos x - \sin x = 0 \Leftrightarrow \sqrt{2} \cos \left(x + \frac{\pi}{4} \right) = 0 \quad *$$

$$\Leftrightarrow \cos \left(x + \frac{\pi}{4} \right) = 0$$

$$\Leftrightarrow x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi; k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \quad ; k \in \mathbb{Z}$$

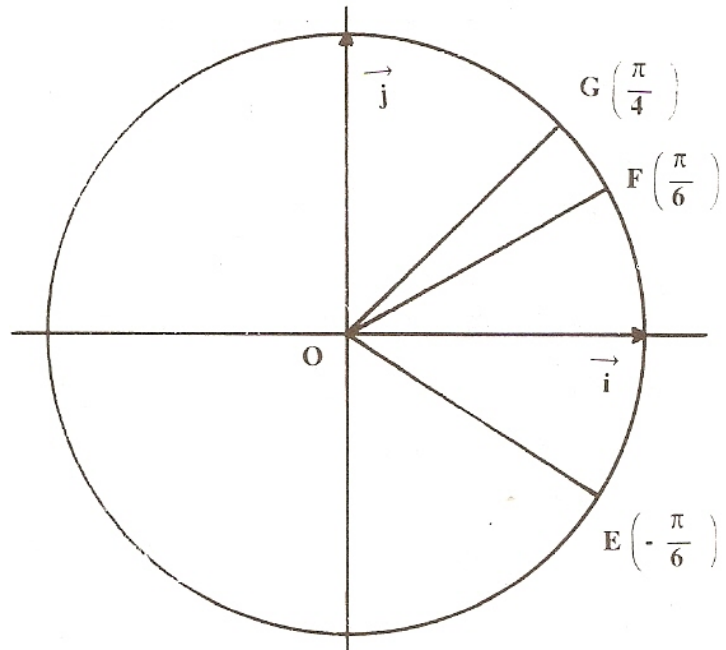
$$\cos x - \frac{\sqrt{3}}{2} = 0 \Leftrightarrow \cos x = \frac{\sqrt{3}}{2} \quad *$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi; k \in \mathbb{Z} \\ \text{أو} \\ x = -\frac{\pi}{6} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

*

$$S = \left\{ \frac{\pi}{4} + k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\}$$

* تمثيل الحلول على الدائرة المتثلثية:



ب- نعتبر $x \in [0, \pi]$

$$A(x) < 0 \Leftrightarrow (\cos x - \sin x) \left(\cos x - \frac{\sqrt{3}}{2} \right) < 0$$

نحل المتراجحتين في المجال $[0, \pi]$:

$$(I_1) : \cos x - \sin x > 0$$

$$(I_2) : \cos x - \frac{\sqrt{3}}{2} > 0$$

$$(I_1) \Leftrightarrow \cos \left(x + \frac{\pi}{4} \right) > 0 *$$

$$X = x + \frac{\pi}{4} \text{ نضع}$$

$$X \in \left[\frac{\pi}{4}, \frac{5\pi}{4} \right] \text{ إذن } x \in [0, \pi]$$

$$\cos \left(x + \frac{\pi}{4} \right) > 0 \Leftrightarrow \cos X > 0 : \text{ وبالتالي فإن}$$

$$\Leftrightarrow X \in \left[\frac{\pi}{4}, \frac{\pi}{2} \right[$$

$$\Leftrightarrow x \in \left[0, \frac{\pi}{4} \right[$$

$$(I_2) \Leftrightarrow \cos x - \frac{\sqrt{3}}{2} > 0 *$$

$$\Leftrightarrow \cos x > \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow x \in \left[0, \frac{\pi}{6} \right[$$

حسب الجدول

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	π
$\cos x - \sin x$	+	+	0	—
$\cos x - \frac{\sqrt{3}}{2}$	+	0	—	—
A (x)	+	0	—	+

وبالتالي فإن : $S = \left] \frac{\pi}{6}, \frac{\pi}{4} \right[$