

(1) * إخطاط $\sin^3 x$

نعلم أن $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$ إذن :

$$\begin{aligned}\sin^3 x &= \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^3 \\ &= \frac{e^{3ix} - 3e^{ix} + 3e^{-ix} - e^{-3ix}}{-8i} \\ &= \frac{(e^{3ix} - e^{-3ix}) - 3(e^{ix} - e^{-ix})}{-8i} \\ &= \frac{2i \sin 3x - 6i \sin x}{-8i} \\ &= -\frac{1}{4} \sin 3x + \frac{3}{4} \sin x\end{aligned}$$

$$\sin^3 x = -\frac{1}{4} \sin 3x + \frac{3}{4} \sin x \quad \text{ومنہ :}$$

$$I = \int_0^{\frac{\pi}{2}} x \sin^3 x \, dx \quad \text{* حساب التكامل :}$$

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$$= \int_0^{\frac{\pi}{2}} x \left(\frac{3}{4} \sin x - \frac{1}{4} \sin 3x \right) dx$$

$$v'(x) = \frac{3}{4} \sin x - \frac{1}{4} \sin 3x \quad \text{و } u(x) = x \quad \text{نضع}$$

$$v(x) = -\frac{3}{4} \cos x + \frac{1}{12} \cos 3x \quad \text{و } u'(x) = 1 \quad \text{إذن}$$

التكامل I يصبح :

$$I = \int_0^{\frac{\pi}{2}} x \sin^3 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} x \left(\frac{3}{4} \sin x - \frac{1}{4} \sin 3x \right) dx$$

$$= \int_0^{\frac{\pi}{2}} u(x) v'(x) \, dx$$

$$\begin{aligned}
 &= [u(x) v(x)]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} u'(x) v(x) dx \\
 &= \left[x \left(-\frac{3}{4} \cos x + \frac{1}{12} \cos 3x \right) \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \left(-\frac{3}{4} \cos x + \frac{1}{12} \cos 3x \right) dx \\
 &= \left[\frac{3}{4} \sin x - \frac{1}{36} \sin 3x \right]_0^{\frac{\pi}{2}} \\
 &= \frac{7}{9}
 \end{aligned}$$

(2) حساب التكامل $J = \int_0^1 \frac{dx}{(x^2 + 1)^2}$

نضع $x = \tan t$ لدينا: $dx = (1 + \tan^2 t) dt$

إذا كان $x = 0$ فإن $t = 0$ وإذا كان $x = 1$ فإن $t = \frac{\pi}{4}$.
التكامل J يصبح :

$$\begin{aligned}
 J &= \int_0^1 \frac{dx}{(x^2 + 1)^2} = \int_0^{\frac{\pi}{4}} \frac{(1 + \tan^2 t)}{(\tan^2 t + 1)^2} dt \\
 &= \int_0^{\frac{\pi}{4}} \frac{1}{\tan^2 t + 1} dt \\
 &= \int_0^{\frac{\pi}{4}} \cos^2 t dt \\
 &= \int_0^{\frac{\pi}{4}} \frac{1 + \cos 2t}{2} dt \\
 &= \frac{1}{2} \left[t + \frac{1}{2} \sin 2t \right]_0^{\frac{\pi}{4}} \\
 &= \frac{\pi}{8} + \frac{1}{4}
 \end{aligned}$$



مع الشامل