

$$J = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x} dx \quad (1) \text{ حساب التكامل}$$

نضع $u(x) = \cos x$ إذن $u'(x) = -\sin x$

$$J = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x} dx = - \int_0^{\frac{\pi}{3}} \frac{u'(x)}{u(x)} dx \quad \text{ومنه :}$$

$$= - [\ln |u(x)|]_0^{\frac{\pi}{3}}$$

$$= - [\ln |\cos x|]_0^{\frac{\pi}{3}}$$

$$= - \ln \frac{1}{2} + \ln 1$$

$$= \ln 2$$

$$(2) \text{ أ- نعلم أن } \cos^2 x + \sin^2 x = 1$$

$$\text{إذن } \cos^2 x = 1 - \sin^2 x$$

$$\text{أي } \cos^2 x = (1 - \sin x)(1 + \sin x)$$

$$\frac{\cos x}{1 + \sin x} = \frac{1 - \sin x}{\cos x} \quad \text{وبالتالي :}$$

ب- حساب $J - I$.

نضع $v'(x) = \cos x$ إذن $v(x) = 1 + \sin x$

$$I - J = \int_0^{\frac{\pi}{3}} \frac{1}{\cos x} dx - \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x} dx \quad \text{لدينا :}$$

$$= \int_0^{\frac{\pi}{3}} \left(\frac{1 - \sin x}{\cos x} \right) dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\cos x}{1 + \sin x} dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{v'(x)}{v(x)} dx$$

$$= [\ln |v(x)|]_0^{\frac{\pi}{3}}$$

$$= [\ln |1 + \sin x|]_0^{\frac{\pi}{3}}$$

$$= \ln \left(1 + \frac{\sqrt{3}}{2} \right)$$

(3) رأينا من خلال الأسئلة السابقة أن :

$$I - J = \ln \left(1 + \frac{\sqrt{3}}{2} \right) \quad \text{و} \quad J = \ln 2$$

إذن بجمع J و $I - J$ نجد :

$$I = \ln 2 + \ln \left(1 + \frac{\sqrt{3}}{2} \right) = \ln (2 + \sqrt{3})$$

