

$$I = \int_0^1 \left(x \ln(x+1) + \frac{1}{x^2 + 2x + 2} \right) dx \quad \text{نضع}$$

$$I_2 = \int_0^1 \frac{1}{x^2 + 2x + 2} dx \quad \text{و} \quad I_1 = \int_0^1 x \ln(x+1) dx$$

نحسب في البداية I_1 و I_2 :

- حساب I_1 :

$$\begin{cases} u'(x) = \frac{1}{x+1} \\ v(x) = \frac{x^2}{2} \end{cases} \quad \text{نضع} \quad \begin{cases} u(x) = \ln(x+1) \\ v'(x) = x \end{cases} \quad \text{إذن}$$

التكامل I_1 يصبح :

$$I_1 = \int_0^1 x \ln(x+1) dx$$

$$= \int_0^1 v'(x) u(x) dx$$

$$= [u(x) v(x)]_0^1 - \int_0^1 v(x) u'(x) dx$$

$$= \left[\frac{x^2}{2} \ln(x+1) \right]_0^1 - \int_0^1 \frac{x^2}{2(x+1)} dx$$

$$= \left[\frac{x^2}{2} \ln(x+1) \right]_0^1 - \frac{1}{2} \int_0^1 \left(x - 1 + \frac{1}{x+1} \right) dx$$

$$\begin{aligned} &= \left[\frac{x^2}{2} \ln(x+1) \right]_0^1 - \frac{1}{2} \left[\frac{x^2}{2} - x + \ln(x+1) \right]_0^1 \\ &= \frac{1}{4} \end{aligned}$$

- حساب I_2 :

$$I_2 = \int_0^1 \frac{dx}{x^2 + 2x + 2} = \int_0^1 \frac{dx}{(x+1)^2 + 1}$$

نضع $t = x + 1$ إذن $dt = dx$.

إذا كان $x = 0$ فإن $t = 1$ وإذا كان $x = 1$ فإن $t = 2$

التكامل I_2 يصبح :

$$\begin{aligned} I_2 &= \int_1^2 \frac{dt}{t^2 + 1} = [\text{Arctan } t]_1^2 \\ &= \text{Arctan } 2 - \text{Arctan } 1 \\ &= \text{Arctan } 2 - \frac{\pi}{4} \end{aligned}$$

أخيرا :

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{1}{4} - \frac{\pi}{4} + \text{Arctan } 2 \\ &= \frac{1 - \pi}{4} + \text{Arctan } 2 \end{aligned}$$

