

a(1) لكل $x \in \mathbb{R}$ لدينا:

$$\begin{aligned} 2 \cos\left(x + \frac{\pi}{4}\right) \sin\left(x + \frac{\pi}{4}\right) &= \sin\left(2\left(x + \frac{\pi}{4}\right)\right) \\ &= \sin\left(2x + \frac{\pi}{2}\right) \\ &= \cos 2x \quad \text{إذن:} \end{aligned}$$

b نعتبر $x \in \mathbb{R}$:

$$4 \cos\left(x + \frac{\pi}{4}\right) \sin\left(x + \frac{\pi}{4}\right) = 1 \Leftrightarrow 2 \cos\left(x + \frac{\pi}{4}\right) \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$$

$$\Leftrightarrow \cos 2x = \frac{1}{2}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{3} + 2k\pi; k \in \mathbb{Z} \\ \text{أو} \\ 2x = -\frac{\pi}{3} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k\pi; k \in \mathbb{Z} \\ \text{أو} \\ x = -\frac{\pi}{6} + k\pi; k \in \mathbb{Z} \end{cases}$$

وبالتالي فإن: $S = \left\{ \frac{\pi}{6} + k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{6} + k\pi / k \in \mathbb{Z} \right\}$

a(2) نعتبر $x \in \mathbb{R}$.

$$\begin{aligned} (2 + \cos x)(4 \cos^2 x - 3) &= 8 \cos^2 x - 6 + 4 \cos^3 x - 3 \cos x \\ &= 4 \cos^3 x + 8 \cos^2 x - 3 \cos x - 6 \\ &= A(x) \end{aligned}$$

b لكل $x \in \mathbb{R}$: $\cos x \geq -1$

إذن: $2 + \cos x \geq 1$.

ومنه $2 + \cos x > 0$

c $A(x) = (2 + \cos x)(4 \cos^2 x - 3)$

و $2 + \cos x > 0$

وبالتالي فإن: $A(x) = 0 \Leftrightarrow 4 \cos^2 x - 3 = 0$

$$\Leftrightarrow 4 \left(\frac{1 + \cos 2x}{2} \right) - 3 = 0$$

$$\Leftrightarrow 2(1 + \cos 2x) - 3 = 0$$

$$\Leftrightarrow \cos 2x = \frac{1}{2}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{3} + 2k\pi; k \in \mathbb{Z} \\ \text{أو} \\ 2x = -\frac{\pi}{3} + k\pi; k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k\pi; k \in \mathbb{Z} \\ \text{أو} \\ x = -\frac{\pi}{6} + k\pi; k \in \mathbb{Z} \end{cases}$$

$$S = \left\{ \frac{\pi}{6} + k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{6} + k\pi / k \in \mathbb{Z} \right\} : \text{ إذن}$$

d- نعتبر $x \in \mathbb{R}$.

$$A(x) > 0 \Leftrightarrow (2 + \cos x)(4 \cos^2 x - 3) > 0$$

$$4 \cos^2 x - 3 = 2 \cos 2x - 1 \quad \text{و} \quad 2 + \cos x > 0 \quad \text{وحيث}$$

$$A(x) > 0 \Leftrightarrow 2 \cos 2x - 1 > 0 \quad \text{إذن}$$

$$\Leftrightarrow \cos 2x > \frac{1}{2}$$

$$2x \in [0, 2\pi] \quad \text{فإن} \quad x \in [0, \pi]$$

وحسب الدائرة المثلثية:

$$\cos 2x > \frac{1}{2} \Leftrightarrow 2x \in \left[0, \frac{\pi}{3} \right[\cup \left] \frac{5\pi}{3}, 2\pi \right]$$

$$\Leftrightarrow x \in \left[0, \frac{\pi}{6} \right[\cup \left] \frac{5\pi}{6}, \pi \right]$$

$$S = \left[0, \frac{\pi}{6} \right[\cup \left] \frac{5\pi}{6}, \pi \right] : \text{ إذن}$$