

نعلم أن $\frac{1}{r+1} x^{r+1}$ هي دالة أصلية للدالة $x \mapsto x^r$ على $]0, +\infty[$ ($r \in \mathbb{Q} - \{-1\}$)

$$\begin{aligned} K_1 &= \int_1^4 \sqrt{x} \, dx \quad * \\ &= \int_1^4 x^{\frac{1}{2}} \, dx \\ &= \left[\frac{1}{\frac{1}{2} + 1} x^{\frac{1}{2} + 1} \right]_1^4 \\ &= \left[\frac{2}{3} x \sqrt{x} \right]_1^4 \\ &= \frac{14}{3} \end{aligned}$$

$$\begin{aligned}
 K_2 &= \int_1^8 \sqrt[3]{x} \, dx & * \\
 &= \int_1^8 x^{\frac{1}{3}} \, dx \\
 &= \left[\frac{1}{\frac{1}{3} + 1} x^{\frac{1}{3} + 1} \right]_1^8 \\
 &= \left[\frac{3}{4} x \cdot \sqrt[3]{x} \right]_1^8 \\
 &= \frac{45}{4}
 \end{aligned}$$

$$\begin{aligned}
 K_3 &= \int_1^8 (1 + \sqrt[3]{x})^2 \, dx & * \\
 &= \int_1^8 (1 + 2\sqrt[3]{x} + \sqrt[3]{x^2}) \, dx \\
 &= \int_1^8 (1 + 2 \cdot x^{\frac{1}{3}} + x^{\frac{2}{3}}) \, dx \\
 &= \left[x + \frac{3}{2} x^{\frac{4}{3}} + \frac{3}{5} x^{\frac{5}{3}} \right]_1^8 \\
 &= \left[x + \frac{3}{2} x \cdot \sqrt[3]{x} + \frac{3}{5} \cdot x \cdot (\sqrt[3]{x})^2 \right]_1^8 \\
 &= \frac{111}{2}
 \end{aligned}$$

$$\begin{aligned}
 K_4 &= \int_1^4 \sqrt{x} (x^2 + x + 1) \, dx & * \\
 &= \int_1^4 \left(x^{\frac{5}{2}} + x^{\frac{3}{2}} + x^{\frac{1}{2}} \right) \, dx \\
 &= \left[\frac{2}{7} x^{\frac{7}{2}} + \frac{2}{5} x^{\frac{5}{2}} + \frac{2}{3} x^{\frac{3}{2}} \right]_1^4 = \frac{254}{7} + \frac{62}{5} + \frac{14}{3}
 \end{aligned}$$

$$\begin{aligned}
 K_5 &= \int_1^2 \left(x^{-\frac{2}{5}} + x^{\frac{1}{3}} \right) \, dx & * \\
 &= \left[\frac{1}{-\frac{2}{5} + 1} x^{-\frac{2}{5} + 1} + \frac{1}{\frac{1}{3} + 1} x^{\frac{1}{3} + 1} \right]_1^2 \\
 &= \left[\frac{5}{3} \cdot x^{\frac{3}{5}} + \frac{3}{4} x^{\frac{4}{3}} \right]_1^2 \\
 &= \frac{5}{3} \cdot 2^{\frac{3}{5}} + \frac{3}{2} \cdot 2^{\frac{1}{3}} - \frac{5}{3} - \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 K_6 &= \int_1^2 (\sqrt[3]{x} - \sqrt{x}) x^{\frac{2}{3}} \, dx & * \\
 &= \int_1^2 \left(x^{\frac{1}{3}} - x^{\frac{1}{2}} \right) x^{\frac{2}{3}} \, dx \\
 &= \int_1^2 \left(x - x^{\frac{7}{6}} \right) \, dx \\
 &= \left[\frac{x^2}{2} - \frac{1}{\frac{7}{6} + 1} x^{\frac{7}{6} + 1} \right]_1^2
 \end{aligned}$$

$$\begin{aligned} &= \left[\frac{x^2}{2} - \frac{6}{13} x^{\frac{13}{6}} \right]_1^2 \\ &= \frac{3}{2} - \frac{6}{13} \left(2^{\frac{13}{6}} - 1 \right) \end{aligned}$$

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