

نعلم أن $(r \in \mathbb{Q} - \{-1\}) \times a \rightarrow \frac{1}{a} \cdot \frac{1}{r+1} (ax+b)^{r+1}$ هي دالة
أصلية للدالة $x \mapsto (ax+b)^r$ على مجال تكون فيه $ax+b > 0$.

$$\begin{aligned} I_1 &= \int_{-1}^1 (x-2)^3 dx & * \\ &= \left[\frac{1}{4} (x-2)^4 \right]_{-1}^1 \\ &= -20 \end{aligned}$$

$$\begin{aligned} I_2 &= \int_{-1}^2 \sqrt{x+2} dx & * \\ &= \int_{-1}^2 (x+2)^{\frac{1}{2}} dx \\ &= \left[\frac{1}{\frac{1}{2}+1} (x+2)^{\frac{1}{2}+1} \right]_{-1}^2 \\ &= \left[\frac{2}{3} (x+2) \sqrt{x+2} \right]_{-1}^2 \\ &= \frac{14}{3} \end{aligned}$$

$$\begin{aligned} I_3 &= \int_1^6 \sqrt[3]{2-x} dx & * \\ &= \int_1^6 (2-x)^{\frac{1}{3}} dx \\ &= \left[\frac{1}{-\frac{1}{3}} \cdot \frac{1}{\frac{1}{3}+1} (2-x)^{\frac{1}{3}+1} \right]_1^6 \\ &= \left[-\frac{3}{4} \cdot (2-x) \cdot \sqrt[3]{2-x} \right]_1^6 \\ &= -\frac{45}{4} \end{aligned}$$

$$\begin{aligned} I_4 &= \int_0^4 \sqrt{2x+1} dx & * \\ &= \int_0^4 (2x+1)^{\frac{1}{2}} dx \\ &= \left[\frac{1}{2} \cdot \frac{1}{\frac{1}{2}+1} (2x+1)^{\frac{1}{2}+1} \right]_0^4 \\ &= \left[\frac{1}{3} \cdot (2x+1) \sqrt{2x+1} \right]_0^4 \\ &= \frac{26}{3} \end{aligned}$$

$$\begin{aligned} I_5 &= \int_0^4 \frac{dx}{\sqrt{2x+1}} & * \\ &= \int_0^4 (2x+1)^{-\frac{1}{2}} dx \\ &= \left[\frac{1}{2} \cdot \frac{1}{-\frac{1}{2}+1} (2x+1)^{-\frac{1}{2}+1} \right]_0^4 \\ &= \left[\sqrt{2x+1} \right]_0^4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} I_6 &= \int_1^2 \frac{dx}{(2x+1)^2} & * \\ &= \int_1^2 (2x+1)^{-2} dx \\ &= \left[\frac{1}{2} \cdot \frac{1}{-\frac{1}{2}+1} (2x+1)^{-\frac{1}{2}+1} \right]_1^2 \\ &= \left[\frac{-1}{2(2x+1)} \right]_1^2 \\ &= \frac{1}{9} \end{aligned}$$