

نعلم أن $x \mapsto \frac{1}{a} \cos(ax + b)$ هي دالة أصلية للدالة $x \mapsto \sin(ax + b)$ على $\mathbb{R}$ و $x \mapsto \frac{1}{a} \sin(ax + b)$ هي دالة أصلية للدالة $x \mapsto \cos(ax + b)$ على $\mathbb{R}$.

$$\begin{aligned} J_1 &= \int_0^{\frac{\pi}{2}} \cos x \sin x \, dx \quad * \\ &= \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin(2x) \, dx \\ &= \frac{1}{2} \left[-\frac{1}{2} \cos(2x) \right]_0^{\frac{\pi}{2}} = \frac{1}{2} \end{aligned}$$

$$J_2 = \int_0^{\frac{\pi}{2}} \sin\left(2x - \frac{\pi}{4}\right) dx \quad *$$

$$= \left[-\frac{1}{2} \cos\left(2x - \frac{\pi}{4}\right) \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$J_3 = \int_{-\frac{\pi}{3}}^{\pi} \cos\left(3x - \frac{\pi}{2}\right) dx \quad *$$

$$= \left[\frac{1}{3} \sin\left(3x - \frac{\pi}{2}\right) \right]_{-\frac{\pi}{3}}^{\pi}$$

$$= \frac{1}{3} \sin\left(3\pi - \frac{\pi}{2}\right) - \frac{1}{3} \sin\left(-\pi - \frac{\pi}{2}\right)$$

$$= \frac{1}{3} \sin \frac{\pi}{2} - \frac{1}{3} \sin\left(-\frac{3\pi}{2}\right)$$

$$= 0$$

$$J_4 = \int_0^{\frac{\pi}{4}} \sin x \cos^3 x \, dx \quad *$$

$$= - \int_0^{\frac{\pi}{4}} (\cos x)' \cos^3 x \, dx$$

$$= \left[-\frac{1}{4} \cos^4 x \right]_0^{\frac{\pi}{4}}$$

$$= \frac{3}{16}$$

$$J_5 = \int_0^{\frac{\pi}{4}} (\tan^3 x + \tan x) \, dx \quad *$$

$$= \int_0^{\frac{\pi}{4}} (\tan^2 x + 1) \tan x \, dx$$

$$= \int_0^{\frac{\pi}{4}} (\tan x)' \tan x \, dx$$

$$= \left[\frac{1}{2} \tan^2 x \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}$$

$$J_6 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x \, dx \quad *$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x} \, dx$$

$$= - \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\cos x)'}{\cos x} \, dx$$

$$= - \left[\ln |\cos x| \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= - \left(\ln\left(\frac{1}{2}\right) - \ln\left(\frac{\sqrt{3}}{2}\right) \right)$$

$$= \frac{1}{2} \ln 3$$

1.

$$J_7 = \int_0^{\frac{\pi}{3}} \tan^2 x \, dx \quad *$$

$$= \int_0^{\frac{\pi}{3}} [(\tan^2 x + 1) - 1] \, dx$$

$$= [\tan x - x]_0^{\frac{\pi}{3}}$$

$$= \sqrt{3} - \frac{\pi}{3}$$

$$J_8 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\tan^2 x + 1}{\tan x} \, dx \quad *$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\tan x)'}{\tan x} \, dx$$

$$= [\ln |\tan x|]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \ln(\sqrt{3}) - \ln\left(\frac{\sqrt{3}}{3}\right)$$

$$= \ln 3$$

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