

$$\begin{aligned} J_1 &= \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin^3 x \, dx & * \\ &= \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin^2 x \sin x \, dx \\ &= \int_0^{\frac{\pi}{2}} \cos^2 x (1 - \cos^2 x) \sin x \, dx \\ &= \int_0^{\frac{\pi}{2}} (\cos^2 x \sin x - \cos^4 x \sin x) \, dx \\ &= \left[-\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x \right]_0^{\frac{\pi}{2}} \\ &= \frac{1}{3} - \frac{1}{5} = \frac{2}{15} \end{aligned}$$

$$\begin{aligned} (\cos x \cdot \sin x)^3 &= \cos^3 x \cdot \sin^3 x & \text{لدينا } * \\ &= \cos^3 x \cdot \sin^2 x \cdot \sin x \\ &= \cos^3 x \cdot (1 - \cos^2 x) \sin x \\ &= \cos^3 x \cdot \sin x - \cos^5 x \cdot \sin x \end{aligned}$$

$$\begin{aligned} J_2 &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos x \cdot \sin x)^3 \, dx & \text{إذن :} \\ &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos^3 x \cdot \sin x - \cos^5 x \sin x) \, dx \\ &= \left[-\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\ &= \frac{1}{16} - \frac{1}{48} = \frac{1}{24} \end{aligned}$$

$$\begin{aligned} (\cos x + \sin x)^2 &= \cos^2 x + \sin^2 x + 2 \sin x \cos x \\ &= 1 + \sin(2x) \end{aligned}$$

* لدينا

$$J_3 = \int_0^{\frac{\pi}{6}} (\cos x + \sin x)^2 dx \quad \text{إذن :}$$

$$= \int_0^{\frac{\pi}{6}} [1 + \sin(2x)] dx$$

$$= \left[x - \frac{1}{2} \cos(2x) \right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{6} - \frac{1}{2} \cos\left(\frac{\pi}{3}\right) + \frac{1}{2} \cos(0)$$

$$= \frac{\pi}{6} + \frac{1}{4}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2} \quad \text{* لدينا}$$

$$J_4 = \int_0^{\frac{\pi}{3}} \cos^2 x dx = \int_0^{\frac{\pi}{3}} \frac{1 + \cos(2x)}{2} dx \quad \text{إذن :}$$

$$= \left[\frac{1}{2} x + \frac{1}{4} \sin(2x) \right]_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{8}$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \quad \text{* لدينا}$$

$$J_5 = \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \text{إذن :}$$

$$= \int_0^{\frac{\pi}{2}} \frac{1 - \cos(2x)}{2} dx$$

$$= \left[\frac{1}{2} x - \frac{1}{4} \sin(2x) \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4}$$

* لنحط $\cos^3 x$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \text{* لدينا}$$

$$\begin{aligned} (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ &= (a^3 + b^3) + 3ab(a + b) \end{aligned} \quad \text{و}$$

$$\cos^3 x = \frac{(e^{ix} + e^{-ix})^3}{2^3} \quad \text{إذن :}$$

$$= \frac{1}{8} [(e^{3ix} + e^{-3ix}) + 3e^{ix} \cdot e^{-ix} (e^{ix} + e^{-ix})]$$

$$= \frac{1}{8} (2 \cos 3x + 3 \cdot 1 \cdot 2 \cos x)$$

$$= \frac{1}{4} \cos(3x) + \frac{3}{4} \cos x$$

$$J_6 = \int_{\frac{\pi}{2}}^{\pi} \cos^3 x dx \quad \text{ومنه فإن :}$$

$$= \int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{4} \cos(3x) + \frac{3}{4} \cos x \right) dx$$

$$= \left[\frac{1}{12} \sin(3x) + \frac{3}{4} \sin x \right]_{-\frac{\pi}{2}}^{\pi}$$

$$= + \frac{1}{12} - \frac{3}{4} = - \frac{2}{3}$$

* لنخطط $\sin^4 x$:

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \text{لدينا :}$$

$$\begin{aligned} (a - b)^4 &= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \\ &= (a^4 + b^4) - 4ab(a^2 + b^2) + 6a^2b^2 \end{aligned}$$

$$\begin{aligned} \sin^4 x &= \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^4 \\ &= \frac{(e^{4ix} + e^{-4ix}) - 4e^{ix} \cdot e^{-ix}(e^{2ix} + e^{-2ix}) + 6(e^{ix} \cdot e^{-ix})^2}{(2i)^4} \\ &= \frac{2 \cos(4x) - 4 \cdot 2 \cos(2x) + 6}{16} \\ &= \frac{1}{8} \cos(4x) - \frac{1}{2} \cos(2x) + \frac{3}{8} \end{aligned}$$

ومنه فإن :

$$\begin{aligned} J_7 &= \int_0^{\frac{\pi}{2}} \sin^4 x \, dx \\ &= \int_0^{\frac{\pi}{2}} \left[\frac{1}{8} \cos(4x) - \frac{1}{2} \cos(2x) + \frac{3}{8} \right] dx \\ &= \left[\frac{1}{32} \sin(4x) - \frac{1}{4} \sin(2x) + \frac{3}{8} x \right]_0^{\frac{\pi}{2}} \\ &= \frac{3\pi}{16} \end{aligned}$$

$$\begin{aligned} \cos^2 x \cdot \sin^2 x &= \left[\frac{1}{2} \sin(2x) \right]^2 \quad \text{* لدينا :} \\ &= \frac{1}{4} \sin^2(2x) \\ &= \frac{1}{4} \frac{1 - \cos(4x)}{2} \end{aligned}$$

$$\begin{aligned} J_8 &= \int_0^{\frac{\pi}{2}} \cos^2 x \sin^2 x \, dx \quad \text{إذن :} \\ &= \int_0^{\frac{\pi}{2}} \left[\frac{1}{8} - \frac{1}{8} \cos(4x) \right] dx \\ &= \left[\frac{1}{8} x - \frac{1}{32} \sin(4x) \right]_0^{\frac{\pi}{2}} \\ &= \frac{\pi}{16} \end{aligned}$$

إعداد الأستاذ : محمد أنفلوس