

* لنحسب $\lim_{x \rightarrow 1} \frac{e^x - e}{x - 1}$

نعلم أن الدالة $x \mapsto e^x$ قابلة للاشتقاق على $\mathbb{R}$

و $\forall x \in \mathbb{R} \quad (e^x)' = e^x$

إذن $\lim_{x \rightarrow a} \frac{e^x - e^a}{x - a} = e^a$

ومنه فإن $\lim_{x \rightarrow 1} \frac{e^x - e}{x - 1} = e^1 = e$

* لنحسب $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{e^x - 1}{\sqrt{x}}$

لدينا $\forall x > 0 \quad \frac{e^x - 1}{\sqrt{x}} = \frac{e^x - 1}{x} \cdot \sqrt{x}$

بما أن $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \sqrt{x} = 0$ و $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$

فإن $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{e^x - 1}{\sqrt{x}} = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(\frac{e^x - 1}{x} \right) \cdot \sqrt{x} = 0$

* لنحسب $\lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x}$

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x} &= \lim_{x \rightarrow 0} \frac{e^x (e^x - 1)}{x} \\ &= \lim_{x \rightarrow 0} \frac{e^x - 1}{x} \cdot e^x\end{aligned}$$

$$\boxed{\lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x} = 1} \quad \text{إذن}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 + x} \quad \text{لنحسب}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 + x} = \lim_{x \rightarrow 0} \frac{e^x - 1}{x} \cdot \frac{1}{x + 1} = 1$$

$$\boxed{\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 + x} = 1} \quad \text{إذن}$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x^2 + 1} \quad \text{* لنحسب}$$

$$\frac{e^x}{x^2 + 1} = \frac{e^x}{x^2} \cdot \frac{1}{1 + \frac{1}{x^2}} \quad \text{لدينا}$$

$$\left. \begin{aligned}\lim_{x \rightarrow +\infty} \frac{e^x}{x^2} &= +\infty \\ \lim_{x \rightarrow +\infty} \frac{1}{1 + \frac{1}{x^2}} &= 1\end{aligned} \right\} \Rightarrow \boxed{\lim_{x \rightarrow +\infty} \frac{e^x}{x^2 + 1} = +\infty}$$

$$\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} \quad \text{* لنحسب}$$

$$\frac{e^{\sin x} - 1}{x} = \frac{e^{\sin x} - 1}{\sin x} \cdot \frac{\sin x}{x} \quad \text{لدينا}$$

$$\left. \begin{aligned}\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{\sin x} &= \lim_{t \rightarrow 0} \frac{e^t - 1}{t} = 1 \\ \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1\end{aligned} \right\}$$

$$\Rightarrow \boxed{\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} = 1}$$

$$\lim_{x \rightarrow +\infty} \left(2x + \frac{e^x}{e^x - 1} \right) \quad \text{* لنحسب}$$

$$\lim_{x \rightarrow +\infty} (2x) = +\infty \quad \text{لدينا :}$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{e^x - 1} = \lim_{x \rightarrow +\infty} \frac{1}{1 - \frac{1}{e^x}} = 1.$$

$$\boxed{\lim_{x \rightarrow +\infty} \left(2x + \frac{e^x}{e^x - 1} \right) = +\infty} \quad \text{إذن :}$$

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(2x + \frac{e^x}{e^x - 1} \right) \quad \text{* لنحسب}$$

لدينا : $\lim_{x \rightarrow 0} (e^x - 1) = 0$ و $\lim_{x \rightarrow 0} e^x = 1$

و $\forall x > 0 \quad e^x - 1 > 0$

إذن $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{e^x}{e^x - 1} = +\infty$

ومنه فإن : $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(2x + \frac{e^x}{e^x - 1} \right) = +\infty$

* لنحسب $\lim_{x \rightarrow +\infty} x^2 e^{\frac{1}{x} - 1}$

$\lim_{x \rightarrow +\infty} \left(\frac{1}{x} - 1 \right) = -1 \Rightarrow \lim_{x \rightarrow +\infty} e^{\frac{1}{x} - 1} = e^{-1}$

$\Rightarrow \lim_{x \rightarrow +\infty} x^2 e^{\frac{1}{x} - 1} = +\infty$

(لأن $e^{-1} > 0$ و $\lim_{x \rightarrow +\infty} x^2 = +\infty$)

* لنحسب $\lim_{\substack{x \rightarrow 0 \\ x > 0}} x^2 e^{\frac{1}{x} - 1}$

نضع $t = \frac{1}{x}$

$\lim_{\substack{x \rightarrow 0 \\ x > 0}} x^2 e^{\frac{1}{x} - 1} = \lim_{t \rightarrow +\infty} \frac{e^{t-1}}{t^2}$

$= \lim_{t \rightarrow +\infty} \frac{1}{e} \cdot \frac{e^t}{t^2} = +\infty$

$\lim_{\substack{x \rightarrow 0 \\ x > 0}} x^2 e^{\frac{1}{x} - 1} = +\infty$

* لنحسب $\lim_{\substack{x \rightarrow 0 \\ x < 0}} x^2 e^{\frac{1}{x} - 1}$

$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{1}{x} - 1 = -\infty \Rightarrow \lim_{\substack{x \rightarrow 0 \\ x < 0}} e^{\frac{1}{x} - 1} = 0$

$\Rightarrow \lim_{\substack{x \rightarrow 0 \\ x < 0}} x^2 e^{\frac{1}{x} - 1} = 0$

* لنحسب $\lim_{x \rightarrow +\infty} (x^2 e^{-x} + x^3)$

لدينا : $x^2 e^{-x} + x^3 = x^2 (e^{-x} + x)$

$\lim_{x \rightarrow +\infty} e^{-x} = 0 \Rightarrow \lim_{x \rightarrow +\infty} (e^{-x} + x) = +\infty$

$\Rightarrow \lim_{x \rightarrow +\infty} (x^2 e^{-x} + x^3) = +\infty$

* لنحسب $\lim_{x \rightarrow -\infty} (x^2 e^{-x} + x^3)$

لدينا : $x^2 e^{-x} + x^3 = x^3 \left(-\frac{e^{-x}}{-x} + 1 \right)$

$\lim_{x \rightarrow -\infty} \frac{e^{-x}}{-x} = \lim_{t \rightarrow +\infty} \frac{e^t}{t} = +\infty$

$\Rightarrow \lim_{x \rightarrow -\infty} x^3 \left(-\frac{e^{-x}}{-x} + 1 \right) = +\infty$

$$\boxed{\lim_{x \rightarrow -\infty} (x^2 e^{-x} + x^3) = +\infty}$$

* لنحسب $\lim_{x \rightarrow +\infty} e^{\frac{1}{x}} \ln x$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} e^{\frac{1}{x}} = e^0 = 1 \\ \lim_{x \rightarrow +\infty} \ln x = +\infty \end{array} \right\} \Rightarrow \boxed{\lim_{x \rightarrow +\infty} e^{\frac{1}{x}} \ln x = +\infty}$$

* لنحسب $\lim_{|x| \rightarrow +\infty} x^2 \left(e^{\frac{1}{x}} - e^{\frac{1}{x+1}} \right)$

لدينا :

$$\begin{aligned} \forall x \in \mathbb{R}^* - \{-1\} \quad x^2 \left(e^{\frac{1}{x}} - e^{\frac{1}{x+1}} \right) &= x^2 e^{\frac{1}{x+1}} \left(e^{\frac{1}{x} - \frac{1}{x+1}} - 1 \right) \\ &= x^2 e^{\frac{1}{x+1}} \left(e^{\frac{1}{x(x+1)}} - 1 \right) \\ &= \frac{x^2}{x(x+1)} \cdot e^{\frac{1}{x+1}} \cdot \frac{e^{\frac{1}{x(x+1)}} - 1}{\frac{1}{x(x+1)}} \end{aligned}$$

ولدينا : $\lim_{|x| \rightarrow +\infty} \frac{x^2}{x(x+1)} = 1$

و $\lim_{|x| \rightarrow +\infty} e^{\frac{1}{x+1}} = e^0 = 1$

و $\lim_{|x| \rightarrow +\infty} \frac{e^{\frac{1}{x(x+1)}} - 1}{\frac{1}{x(x+1)}} = \lim_{t \rightarrow 0} \frac{e^t - 1}{t} = 1$

$$\boxed{\lim_{|x| \rightarrow +\infty} x^2 \left(e^{\frac{1}{x}} - e^{\frac{1}{x+1}} \right) = 1} \quad \text{إذن}$$

* لنحسب $\lim_{x \rightarrow +\infty} \sqrt{x} \cdot e^{-x^2}$

$$\begin{aligned} \forall x > 0 \quad \sqrt{x} \cdot e^{-x^2} &= \frac{\sqrt{x}}{e^{\sqrt{x}}} \cdot e^{-x^2 + \sqrt{x}} \\ &= \frac{1}{\frac{e^{\sqrt{x}}}{\sqrt{x}}} \cdot e^{-x^2 \left(1 - \frac{1}{\sqrt{x}} \right)} \end{aligned}$$

$$\lim_{x \rightarrow +\infty} \frac{e^{\sqrt{x}}}{\sqrt{x}} = +\infty \Rightarrow \lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{e^{\sqrt{x}}} = 0$$

$$\lim_{x \rightarrow +\infty} -x^2 \left(1 - \frac{1}{\sqrt{x}} \right) = -\infty \Rightarrow \lim_{x \rightarrow +\infty} \left(e^{-x^2 + \sqrt{x}} \right) = 0$$

$$\boxed{\lim_{x \rightarrow +\infty} \sqrt{x} \cdot e^{-x^2} = 0} \quad \text{إذن}$$

* لنحسب $\lim_{x \rightarrow 1} \frac{\sqrt{x}}{x-1} e^{\frac{1}{x-1}}$

$$\frac{\sqrt{x}}{x-1} e^{\frac{1}{x-1}} = \sqrt{x} \left(\frac{1}{x-1} \cdot e^{\frac{1}{x-1}} \right) \quad \text{لدينا :}$$

$$\lim_{\substack{x \rightarrow 1 \\ x > 1}} \frac{1}{x-1} = +\infty \Rightarrow \lim_{x \rightarrow +\infty} \frac{1}{x-1} e^{\frac{1}{x-1}} = +\infty$$

$$\Rightarrow \boxed{\lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{x-1} e^{\frac{1}{x-1}} = +\infty}$$

$$\lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{1}{x-1} = -\infty \Rightarrow \lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{1}{x-1} e^{\frac{1}{x-1}} = \lim_{t \rightarrow -\infty} t e^t = 0$$

$$\Rightarrow \boxed{\lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{\sqrt{x}}{x-1} e^{\frac{1}{x-1}} = 0}$$

$$\lim_{x \rightarrow -\infty} e^{-x} \ln(1 + e^x) \quad * \text{ لنحسب}$$

$$e^{-x} \ln(1 + e^x) = \frac{\ln(1 + e^x)}{e^x} \quad \text{لدينا :}$$

$t = e^x$ نضع

$$\lim_{x \rightarrow -\infty} e^{-x} \ln(1 + e^x) = \lim_{\substack{t \rightarrow 1 \\ t > 0}} \frac{\ln(1 + t)}{t} = 1$$

$$\boxed{\lim_{x \rightarrow -\infty} e^{-x} \ln(1 + e^x) = 1} \quad \text{إذن :}$$

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