

$$\begin{aligned} J_1 &= \int_0^{\frac{\pi}{4}} \frac{\cos x}{2 + \sin x} dx \\ &= \int_0^{\frac{\pi}{4}} \frac{(2 + \sin x)'}{2 + \sin x} dx \\ &= [\ln (2 + \sin x)]_0^{\frac{\pi}{4}} \\ &= \ln \left(2 + \frac{\sqrt{2}}{2} \right) - \ln (2) \\ &= \ln \left(\frac{4 + \sqrt{2}}{4} \right) \end{aligned}$$

$$\begin{aligned} J_2 &= \int_0^{\frac{\pi}{6}} \frac{\sin(2x)}{\sqrt{1 + \sin^2 x}} dx \\ &= \int_0^{\frac{\pi}{6}} \frac{2 \sin x \cos x}{\sqrt{1 + \sin^2 x}} dx \\ &= 2 \int_0^{\frac{\pi}{6}} \frac{(1 + \sin^2 x)'}{2 \sqrt{1 + \sin^2 x}} dx \\ &= 2 \left[\sqrt{1 + \sin^2 x} \right]_0^{\frac{\pi}{6}} \\ &= 2 \left(\sqrt{\frac{5}{4}} - 1 \right) \\ &= \sqrt{5} - 2 \end{aligned}$$

$$\begin{aligned} J_3 &= \int_0^{\pi} (1 + \cos x)^2 \cdot \sin x dx \\ &= - \int_0^{\pi} (1 + \cos x)^2 (1 + \cos x)' dx \\ &= - \left[\frac{(1 + \cos x)^3}{3} \right]_0^{\pi} \\ &= \frac{8}{3} \end{aligned}$$

$$\begin{aligned} J_4 &= \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{(1 + \sin^2 x)^3} dx \\ &= \int_0^{\frac{\pi}{4}} \frac{2 \sin x \cos x}{(1 + \sin^2 x)^3} dx \\ &= \int_0^{\frac{\pi}{4}} (1 + \sin^2 x)' (1 + \sin^2 x)^{-3} dx \\ &= \left[\frac{1}{-3 + 1} (1 + \sin^2 x)^{-3+1} \right]_0^{\frac{\pi}{4}} \\ &= \left[\frac{-1}{2(1 + \sin^2 x)^2} \right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{2} - \frac{2}{9} = \frac{5}{18} \end{aligned}$$

$$\begin{aligned} J_5 &= \int_0^{\frac{\pi}{2}} \frac{3 \sin x}{(1 + \cos x)^2} dx \\ &= 3 \int_0^{\frac{\pi}{2}} \frac{-(1 + \cos x)'}{(1 + \cos x)^2} dx \\ &= 3 \left[\frac{1}{1 + \cos x} \right]_0^{\frac{\pi}{2}} \\ &= 3 \left(1 - \frac{1}{2} \right) \\ &= \frac{3}{2} \end{aligned}$$

$$J_6 = \int_0^{\frac{\pi}{2}} \cos x \cdot \sqrt[3]{1 + \sin x} dx$$

$$\begin{aligned} &= \int_0^{\frac{\pi}{2}} (1 + \sin x)' (1 + \sin x)^{\frac{1}{3}} dx \\ &= \left[\frac{1}{\frac{1}{3} + 1} (1 + \sin x)^{\frac{1}{3} + 1} \right]_0^{\frac{\pi}{2}} \\ &= \left[\frac{3}{4} (1 + \sin x)^{\frac{4}{3}} \right]_0^{\frac{\pi}{2}} \\ &= \frac{3}{4} \cdot \left(2^{\frac{4}{3}} - 1 \right) \end{aligned}$$

إعداد الأستاذ : محمد أنفلوس