

(1) لنحل في IN المعادلة $A_n^2 + C_n^3 = 7$ الشروط $(3 \leq n \text{ و } 2 \leq n)$ أي $3 \leq n$

$$\begin{aligned} A_n^2 + C_n^3 = 7 &\Leftrightarrow n(n-1) + \frac{n(n-1)(n-2)}{6} = 7 \\ &\Leftrightarrow 6(n-1)n + n(n-1)(n-2) = 6 \cdot 7 \\ &\Leftrightarrow (n-1)n(n-2+6) = 6 \cdot 7 \\ &\Leftrightarrow (n-1)n(n+4) = 2 \cdot 3 \cdot 7 \\ &\Leftrightarrow \boxed{n = 3} \end{aligned}$$

$$C_n^3 - C_n^2 = \frac{n^3 - 6n^2 + 25}{6} \quad (2)$$

الشروط : $3 \leq n$

$$\begin{aligned} C_n^3 - C_n^2 = \frac{n^3 - 6n^2 + 25}{6} &\Leftrightarrow \frac{n(n-1)(n-2)}{6} - \frac{n(n-1)}{2} = \frac{n^3 - 6n^2 + 25}{6} \\ &\Leftrightarrow \frac{n(n-1)(n-2) - 3n(n-1)}{6} = \frac{n^3 - 6n^2 + 25}{6} \\ &\Leftrightarrow (n^3 - 3n^2 + 2n) - (3n^2 - 3n) = n^3 - 6n^2 + 25 \\ &\Leftrightarrow n^3 - 6n^2 + 5n = n^3 - 6n^2 + 25 \\ &\Leftrightarrow 5n = 25 \\ &\Leftrightarrow \boxed{n = 5} \end{aligned}$$

$$C_n^5 = 17 C_n^4 \quad (3)$$

الشروط : $n \leq 5$

$$\begin{aligned} C_n^5 = 17 C_n^4 &\Leftrightarrow \frac{n!}{(n-5)! 5!} = 17 \cdot \frac{n!}{(n-4)! 4!} \\ &\Leftrightarrow \frac{1}{(n-5)! 4! 5} = \frac{17}{(n-5)! (n-4)! 4!} \\ &\Leftrightarrow \frac{1}{5} = \frac{17}{n-4} \\ &\Leftrightarrow n-4 = 5 \cdot 17 \\ &\Leftrightarrow \boxed{n = 89} \end{aligned} \quad (4)$$

$$\begin{aligned} 20 \cdot n! = (n+2)! &\Leftrightarrow 20 \cdot n! = (n+2) \cdot (n+1) \cdot n! \\ &\Leftrightarrow (n+2)(n+1) = 20 = 5 \times 4 \\ &\Leftrightarrow \boxed{n = 3} \end{aligned}$$

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