

$$\begin{aligned}K_1 &= \int_1^e \frac{\ln x}{x} dx \\&= \int_1^e (\ln x)' \ln x dx \\&= \left[\frac{1}{2} (\ln x)^2 \right]_1^e \\&= \frac{1}{2} \\K_2 &= \int_e^{e^2} \frac{(\ln x)^2}{x} dx \\&= \int_e^{e^2} (\ln x)' (\ln x)^2 dx \\&= \left[\frac{1}{3} (\ln x)^3 \right]_e^{e^2} \\&= \frac{1}{3} (2^3 - 1^3) \\&= \frac{7}{3} \\K_3 &= \int_e^{e^2} \frac{dx}{x (\ln x)^2} \\&= \int_e^{e^2} \frac{(\ln x)'}{(\ln x)^2} dx \\&= \left[-\frac{1}{\ln x} \right]_e^{e^2} = -\frac{1}{2} \\K_4 &= \int_e^{e^2} \frac{dx}{x \ln x} \\&= \int_e^{e^2} \frac{(\ln x)'}{\ln x} dx \\&= [\ln |\ln x|]_e^{e^2} \\&= \ln(2) \\K_5 &= \int_0^{e-1} \frac{\ln(x+1)}{x+1} dx \\&= \int_0^{e-1} (\ln(x+1))' \ln(x+1) dx \\&= \left[\frac{1}{2} (\ln(x+1))^2 \right]_0^{e-1}\end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} (\ln(e) - \ln(1)) \\ &= \frac{1}{2} \\ K_6 &= \int_1^e \frac{(1 + \ln x)^2}{x} dx \\ &= \int_1^e \frac{1 + 2 \ln x + (\ln x)^2}{x} dx \\ &= \int_1^e \left(\frac{1}{x} + 2 (\ln x)' \ln x + (\ln x)' (\ln x)^2 \right) dx \\ &= \left[\ln x + (\ln x)^2 + \frac{1}{3} (\ln x)^3 \right]_1^e \\ &= 1 + 1 + \frac{1}{3} = \frac{7}{3} \end{aligned}$$

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