

$$\begin{aligned} I_1 &= \int_1^{e^2} \frac{dx}{x(1 + \ln x)^2} & * \\ &= \int_1^{e^2} \frac{(1 + \ln x)'}{(1 + \ln x)^2} dx \\ &= \left[\frac{-1}{1 + \ln x} \right]_1^{e^2} \\ &= -\frac{1}{3} + 1 = \frac{2}{3} \\ I_2 &= \int_1^e \frac{dx}{x(1 + \ln x)} & * \\ &= [\ln |1 + \ln x|]_1^e \\ &= \ln 2 \end{aligned}$$

$$\begin{aligned} I_3 &= \int_1^e \frac{\sqrt[3]{1 + 2 \ln x}}{x} dx \\ &= \frac{1}{2} \int_1^e (1 + 2 \ln x)' (1 + 2 \ln x)^{\frac{1}{3}} dx \\ &= \frac{1}{2} \left[\frac{3}{4} (1 + 2 \ln x)^{\frac{4}{3}} \right]_1^e \\ &= \frac{3}{8} \left(3^{\frac{4}{3}} - 1 \right) \\ &= \frac{3}{8} (3 \cdot \sqrt[3]{3} - 1) \end{aligned}$$

$$\begin{aligned} I_4 &= \int_0^1 \frac{e^x - e^{-x}}{(e^x + e^{-x})^2} dx \\ &= \int_0^1 \frac{(e^x + e^{-x})'}{(e^x + e^{-x})^2} dx \\ &= \left[\frac{-1}{e^x + e^{-x}} \right]_0^1 \\ &= \frac{-1}{e + e^{-1}} + \frac{1}{2} \end{aligned}$$

$$\begin{aligned} I_5 &= \int_0^{\ln 6} e^x \cdot \sqrt{e^x + 3} dx \\ &= \int_0^{\ln 6} (e^x + 3)' (e^x + 3)^{\frac{1}{2}} dx \\ &= \left[\frac{2}{3} (e^x + 3)^{\frac{3}{2}} \right]_0^{\ln 6} \\ &= \frac{2}{3} \left(9^{\frac{3}{2}} - 4^{\frac{3}{2}} \right) \\ &= \frac{2}{3} \cdot (3^3 - 2^3) \\ &= \frac{38}{3} \end{aligned}$$

$$\begin{aligned} I_6 &= \int_{\frac{1}{e}}^e \frac{(2 + \ln x)^{\frac{2}{5}}}{x} dx \\ &= \left[\frac{1}{\frac{2}{5} + 1} (2 + \ln x)^{\frac{2}{5} + 1} \right]_{\frac{1}{e}}^e \\ &= \left[\frac{5}{7} (2 + \ln x)^{\frac{7}{5}} \right]_{\frac{1}{e}}^e \\ &= \frac{5}{7} \left(3^{\frac{7}{5}} - 1 \right) \end{aligned}$$

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