

$$\begin{cases} u_0 = 1 \\ u_n = \frac{u_{n-1} + 5}{3} \quad ; n \in \mathbb{N}^* \\ v_n = \frac{u_n - u_{n-1}}{3} \quad ; n \in \mathbb{N}^* \end{cases}$$

$$v_{n+1} = \frac{u_{n+1} - u_n}{3} \quad (1)$$

$$\frac{\frac{u_n + 5}{3} - \frac{u_{n-1} + 5}{3}}{3}$$

$$= \frac{1}{3} \cdot \frac{u_n - u_{n-1}}{3}$$

$$= \frac{1}{3} v_n$$

$$\boxed{\forall n \in \mathbb{N} \quad ; \quad v_{n+1} = \frac{1}{3} v_n} \quad \text{إذن}$$

ومنه فإن (v_n) متتالية هندسية أساسها $\frac{1}{3}$.

$$v_1 = \frac{u_1 - u_0}{3} = \frac{1}{3} \quad , \quad u_1 = \frac{u_0 + 5}{3} = 2 \quad * (2)$$

$$\forall n \in \mathbb{N} \quad ; \quad v_n = \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{n-1} \quad \text{إذن}$$

$$\boxed{\forall n \in \mathbb{N} \quad ; \quad v_n = \left(\frac{1}{3}\right)^n} \quad \text{أي}$$

$$S_n = v_1 + v_2 + \dots + v_n \quad *$$

$$= \frac{v_1 \left[1 - \left(\frac{1}{3}\right)^n \right]}{1 - \frac{1}{3}}$$

$$S_n = \frac{1}{2} \left[1 - \left(\frac{1}{3} \right)^n \right]$$

إذن

$$+ 3 v_1 = u_1 - u_0$$

$$+ 3 v_2 = u_2 - u_1$$

$$+ 3 v_3 = u_3 - u_2$$

$$+ \vdots$$

$$+ 3 v_n = u_n - u_{n-1}$$

$$\hline 3 (v_1 + v_2 + \dots + v_n) = u_n - u_0$$

$$u_n = 3 S_n + u_0$$

إذن

$$\forall n \in \mathbb{N} \quad ; \quad u_n = 3 S_n + 1$$

أي

$$S_n = \frac{1}{2} \left[1 - \left(\frac{1}{3} \right)^n \right]$$

(4) * بما أن

$$\forall n \in \mathbb{N} \quad ; \quad u_n = \frac{3}{2} \left[1 - \left(\frac{1}{3} \right)^n \right] + 1$$

فإن

$$-1 < \frac{1}{3} < 1$$

* بما أن

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{3} \right)^n = 0$$

فإن

$$\lim_{n \rightarrow +\infty} u_n = \frac{5}{2}$$

ومنه فإن