

$$I_1 = \int_0^{\ln 2} (x^2 + x) e^x dx \quad * \text{ لنحسب :}$$

$$\begin{cases} u'(x) = 2x + 1 \\ v(x) = e^x \end{cases} \quad , \quad \begin{cases} u(x) = u^2 + x \\ v'(x) = e^x \end{cases} \quad * \text{ نضع :}$$

$$I_1 = [(x^2 + x) e^x]_0^{\ln 2} - \int_0^{\ln 2} (2x + 1) e^x dx$$

$$= 2(1 + \ln 2) \ln 2 - \int_0^{\ln 2} (2x + 1) e^x dx$$

$$\begin{cases} u_1'(x) = 2 \\ v(x) = e^x \end{cases} \quad \text{و} \quad \begin{cases} u_1(x) = 2x + 1 \\ v'(x) = e^x \end{cases} \quad \text{نضع :} *$$

$$\int_0^{\ln 2} (2x + 1) e^x dx = [(2x + 1) e^x]_0^{\ln 2} - \int_0^{\ln 2} 2 e^x dx$$

$$= 2(2 \ln 2 + 1) - 1 - [2 e^x]_0^{\ln 2}$$

$$= 4 \ln 2 + 1 - 4 + 2$$

$$\boxed{I_1 = 2(-1 + \ln 2) \ln 2 + 1} \quad \text{إذن :}$$

$$I_2 = \int_0^{\frac{\pi}{2}} (x - 1)^2 \cos x dx \quad \text{لنحسب :} *$$

$$\begin{cases} u'(x) = 2(x - 1) \\ v(x) = \sin x \end{cases} \quad \text{و} \quad \begin{cases} u(x) = (x - 1)^2 \\ v'(x) = \cos x \end{cases} \quad \text{نضع :} *$$

$$I_2 = [(x - 1)^2 \sin x]_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} (x - 1) \sin x dx \quad \text{إذن :}$$

$$= \left(\frac{\pi}{2} - 1\right)^2 - 2 \int_0^{\frac{\pi}{2}} (x - 1) \sin x dx$$

$$\begin{cases} u_1'(x) = 1 \\ v_1(x) = -\cos x \end{cases} \quad \text{و} \quad \begin{cases} u_1(x) = x - 1 \\ v_1'(x) = \sin x \end{cases} \quad \text{نضع :} *$$

إذن :

$$\int_0^{\frac{\pi}{2}} (x - 1) \sin x dx = [-(x - 1) \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= [-(x - 1) \cos x + \sin x]_0^{\frac{\pi}{2}}$$

$$= 1 + (-1) = 0$$

$$\boxed{I_2 = \left(\frac{\pi}{2} - 1\right)^2} \quad \text{إذن :}$$

$$I_3 = \int_0^1 (x^2 + x + 1) \sin(\pi x) dx \quad \text{لنحسب :} *$$

$$\begin{cases} u'(x) = 2x + 1 \\ v(x) = -\frac{1}{\pi} \cos(\pi x) \end{cases} \quad \text{و} \quad \begin{cases} u(x) = x^2 + x + 1 \\ v'(x) = \sin(\pi x) \end{cases} \quad \text{نضع :} *$$

$$I_3 = \left[-\frac{1}{\pi} (x^2 + x + 1) \cos(\pi x)\right]_0^1 + \frac{1}{\pi} \int_0^1 (2x + 1) \cos(\pi x) dx$$

$$= \frac{3}{\pi} + \frac{1}{\pi} + \frac{1}{\pi} \int_0^1 (2x + 1) \cos(\pi x) dx$$

نضع :

$$\begin{cases} u_1'(x) = 2 \\ v_1(x) = \frac{1}{\pi} \sin(\pi x) \end{cases} \quad \text{و} \quad \begin{cases} u_1(x) = 2x + 1 \\ v_1'(x) = \cos(\pi x) \end{cases}$$

$$\int_0^1 (2x + 1) \cos(\pi x) dx = \left[\frac{1}{\pi} (2x + 1) \sin \pi x\right]_0^1 - \frac{2}{\pi} \int_0^1 \sin \pi x dx$$

$$= 0 - \frac{2}{\pi} \left[-\frac{1}{\pi} \cos(\pi x)\right]_0^1 = -\frac{4}{\pi}$$

$$I_3 = \frac{4}{\pi} - \frac{4}{\pi^2} \quad \text{إذن :}$$

$$I_4 = \int_0^{\pi} \cos x \cdot e^x dx \quad \text{نحسب :}$$

$$\begin{cases} u'(x) = -\sin x \\ v(x) = e^x \end{cases} \quad \text{و} \quad \begin{cases} u(x) = \cos x \\ v'(x) = e^x \end{cases} \quad \text{نضع :}$$

$$I_4 = [\cos x \cdot e^x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \sin x \cdot e^x \cdot dx$$
$$= -1 + \int_0^{\frac{\pi}{2}} \sin x \cdot e^x \cdot dx$$

$$\begin{cases} u_1'(x) = \cos x \\ v_1(x) = e^x \end{cases} \quad \text{و} \quad \begin{cases} u_1(x) = \sin x \\ v_1'(x) = e^x \end{cases} \quad \text{نضع :}$$

$$\int_0^{\frac{\pi}{2}} \sin x \cdot e^x \cdot dx = [\sin x \cdot e^x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cos x \cdot e^x \cdot dx$$
$$= e^{\frac{\pi}{2}} - I_4$$

$$I_4 = -1 + e^{\frac{\pi}{2}} - I_4 \quad \text{إذن :}$$

$$I_4 = \frac{1}{2} \left(-1 + e^{\frac{\pi}{2}} \right) \quad \text{ومنه فإن :}$$

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