

(1) لنحدد a و b حيث

$$\forall x \in \mathbb{R}^* - \{1\} ; \frac{1}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1}$$

$$\begin{aligned} \frac{a}{x} + \frac{b}{x-1} &= \frac{a(x-1) + bx}{x(x-1)} \\ &= \frac{(a+b)x - a}{x(x-1)} \end{aligned}$$

$$\frac{1}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1} \Leftrightarrow \begin{cases} a+b=0 \\ -a=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} a=-1 \\ b=1 \end{cases}$$

$$\boxed{\forall x \in \mathbb{R}^* - \{1\} ; \frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}} \quad \text{إذن :}$$

$$\begin{aligned} \int_2^3 \frac{2}{x(x-1)} dx &= 2 \int_2^3 \left(\frac{-1}{x} + \frac{1}{x-1} \right) dx \\ &= 2 [-\ln x + \ln(x-1)]_2^3 \\ &= 2 \left[\ln \left(\frac{x-1}{x} \right) \right]_2^3 \end{aligned}$$

$$\boxed{\int_2^3 \frac{2}{x(x-1)} dx = 2 \ln \left(\frac{4}{3} \right)} \quad \text{إذن :}$$

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