

(1) لكل $x \in \mathbb{R}$:

$$5 \cos 2x + 7 = 5(\cos^2 x - \sin^2 x) + 7(\cos^2 x + \sin^2 x) \\ = 12 \cos^2 x + 2 \sin^2 x$$

(2) لكل $x \in \mathbb{R}$:

$$G(x) = 2 \sin^2 x + 12 \cos^2 x - 10 \sin x \cos x \\ = 5 \cos 2x + 7 - 10 \sin x \cos x \\ = 5 \cos 2x - 5(2 \sin x \cos x) + 7 \\ = 5 \cos 2x - 5 \sin 2x + 7 \\ = 5(\cos 2x - \sin 2x) + 7$$

$$= 5 \left(\sqrt{2} \cos \left(2x + \frac{\pi}{4} \right) \right) + 7$$

$$= 5\sqrt{2} \cos \left(2x + \frac{\pi}{4} \right) + 7$$

$$G(x) = 12 \Leftrightarrow 5\sqrt{2} \cos \left(2x + \frac{\pi}{4} \right) + 7 = 12 \quad (3)$$

$$\Leftrightarrow 5\sqrt{2} \cos \left(2x + \frac{\pi}{4} \right) = 5$$

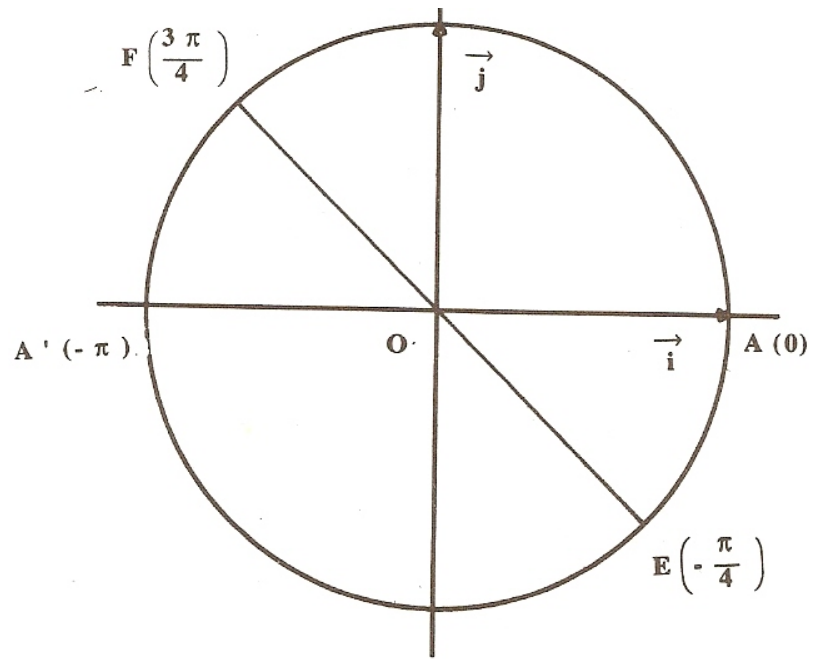
$$\Leftrightarrow \cos \left(2x + \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \begin{cases} 2x + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi; k \in \mathbb{Z} \\ \text{أو} \\ 2x + \frac{\pi}{4} = -\frac{\pi}{4} + 2k\pi; k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = k\pi; k \in \mathbb{Z} \\ \text{أو} \\ x = -\frac{\pi}{4} + k\pi; k \in \mathbb{Z} \end{cases}$$

$$x \in [-\pi, \pi[\Leftrightarrow x \in \left\{ -\pi, -\frac{\pi}{4}, 0, \frac{3\pi}{4} \right\}$$

تمثيل الحلول على الدائرة المثلثية:



(4) حلول المترابحة $G(x) < 7$:

$$G(x) < 7 \Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$X = 2x + \frac{\pi}{4} \text{ نضع}$$

$$X \in [-\pi, \pi] \text{ إذن } x \in \left[-\frac{5\pi}{8}, \frac{3\pi}{8}\right]$$

$$\cos X < 0 \Leftrightarrow X \in \left[-\pi, -\frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}, \pi\right]$$

$$x = \frac{X - \frac{\pi}{4}}{2} \text{ وحيث أن}$$

$$x \in \left[-\frac{5\pi}{8}, -\frac{3\pi}{8}\right] \cup \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \text{ فإن}$$

$$S = \left[-\frac{5\pi}{8}, -\frac{3\pi}{8}\right] \cup \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \text{ وبالتالي فإن:}$$