

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx & J &= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \\
 I + J &= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \quad (1) \\
 &= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx \\
 &= \int_0^{\frac{\pi}{2}} 1. dx \\
 &= [x]_0^{\frac{\pi}{2}} \\
 &\boxed{I + J = \frac{\pi}{2}} \quad \text{إذن :}
 \end{aligned}$$

$$\begin{aligned}
 I - J &= \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{\cos x + \sin x} dx \quad * \\
 &= \int_0^{\frac{\pi}{2}} \frac{(\cos x + \sin x)'}{\cos x + \sin x} dx \\
 &= [\ln |\cos x + \sin x|]_0^{\frac{\pi}{2}} \\
 &\boxed{I - J = 0} \quad \text{إذن :}
 \end{aligned}$$

$$\begin{cases} I + J = \frac{\pi}{2} \\ I - J = 0 \end{cases} \Leftrightarrow \begin{cases} 2I = \frac{\pi}{2} \\ I = J \end{cases} \quad (2)$$

$$\boxed{J = \frac{\pi}{4} \quad \text{و} \quad I = \frac{\pi}{4}} \quad \text{أي}$$

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