

$$I = \int_0^{\frac{\pi}{2}} \cos^4 x \, dx \quad , \quad J = \int_0^{\frac{\pi}{2}} \sin^4 x \, dx \quad , \quad K = \int_0^{\frac{\pi}{2}} \cos^2 x \sin^2 x \, dx$$

$$I - J = \int_0^{\frac{\pi}{2}} (\cos^4 x - \sin^4 x) \, dx \quad (1)$$

$$= \int_0^{\frac{\pi}{2}} (\cos^2 x - \sin^2 x) (\cos^2 x + \sin^2 x) dx$$

$$= \int_0^{\frac{\pi}{2}} \cos(2x) dx$$

$$= \left[\frac{1}{2} \sin(2x) \right]_0^{\frac{\pi}{2}}$$

$$\boxed{I - J = 0}$$

: إذن

$$I + J + 2K = \int_0^{\frac{\pi}{2}} (\cos^4 x + \sin^4 x + 2 \sin^2 x \cos^2 x) dx$$

$$= \int_0^{\frac{\pi}{2}} (\cos^2 x + \sin^2 x)^2 dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$\boxed{I + J + 2K = \frac{\pi}{2}}$$

: إذن

$$I + J - 2K = \int_0^{\frac{\pi}{2}} (\cos^4 x + \sin^4 x - 2 \cos^2 x \sin^2 x) dx$$

$$= \int_0^{\frac{\pi}{2}} (\cos^2 x - \sin^2 x)^2 dx$$

$$= \int_0^{\frac{\pi}{2}} \cos^2(2x) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1 + \cos(4x)}{2} dx$$

$$= \left[\frac{1}{2} x + \frac{1}{8} \sin(4x) \right]_0^{\frac{\pi}{2}}$$

$$\boxed{I + J - 2K = \frac{\pi}{4}}$$

: إذن

$$\begin{cases} I - J = 0 \\ I + J + 2K = \frac{\pi}{2} \\ I + J - 2K = \frac{\pi}{4} \end{cases} \Leftrightarrow \begin{cases} I = J \\ 2I + 2K = \frac{\pi}{2} \\ 2I - 2K = \frac{\pi}{4} \end{cases} \quad (2)$$

$$\Leftrightarrow \begin{cases} I = J \\ I + K = \frac{\pi}{4} \\ I - K = \frac{\pi}{8} \end{cases}$$

$$\Leftrightarrow \begin{cases} I = J \\ 2I = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8} \\ 2K = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8} \end{cases}$$

$$\boxed{K = \frac{\pi}{16} \quad , \quad I = J = \frac{3\pi}{16}} \quad : \text{إذن}$$