

$$u(x) = x + \sqrt{1+x^2}$$

$$\forall x \in \mathbb{R} \quad u'(x) = 1 + \frac{x}{\sqrt{1+x^2}} \quad (1)$$

$$= \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2}}$$

$$I_1 = \int_0^1 \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2}} dx \quad (2)$$

$$= \int_0^1 u'(x) dx$$

$$= [u(x)]_0^1$$

$$I_1 = \sqrt{2}$$

إذن :

لدينا :

$$\frac{1}{\sqrt{1+x^2} (x + \sqrt{1+x^2})^2} = \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2} (x + \sqrt{x^2+1})^3}$$

$$= \frac{u'(x)}{u^3(x)}$$

$$I_2 = \int_0^1 \frac{dx}{\sqrt{1+x^2} (x + \sqrt{1+x^2})^2}$$

$$= \int_0^1 \frac{u'(x)}{u^3(x)} dx$$

$$= \left[\frac{-1}{2u^2(x)} \right]_0^1$$

$$I_2 = \frac{-1}{2(1+\sqrt{2})^2} + \frac{1}{2} = \frac{1}{1+\sqrt{2}}$$

إذن :

لدينا :

$$\frac{1}{\sqrt{1+x^2}} = \frac{x + \sqrt{x^2+1}}{\sqrt{1+x^2} (x + \sqrt{x^2+1})}$$

$$= \frac{u'(x)}{u(x)}$$

$$I_3 = \int_0^1 \frac{1}{\sqrt{1+x^2}} dx$$

إذن :

$$= \int_0^1 \frac{u'(x)}{u(x)} dx$$

$$= [\ln |u(x)|]_0^1$$

$$I_3 = \ln(1+\sqrt{2})$$

إذن :

$$\frac{\sqrt{x + \sqrt{1+x^2}}}{\sqrt{1+x^2}} = \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2}} \times \frac{1}{\sqrt{x + \sqrt{1+x^2}}}$$

$$= \frac{u'(x)}{\sqrt{u(x)}}$$

$$\begin{aligned} I_4 &= \int_0^1 \frac{\sqrt{x + \sqrt{1 + x^2}}}{\sqrt{1 + x^2}} dx && \text{إذن :} \\ &= \int_0^1 \frac{u'(x)}{\sqrt{u(x)}} dx \\ &= \left[2 \sqrt{u(x)} \right]_0^1 \end{aligned}$$

$$I_4 = 2 \sqrt{1 + \sqrt{2}} - 2 \quad \text{إذن :}$$

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