

$$I = \int_0^1 \frac{dx}{\sqrt{x^2 + 2}} \quad , \quad J = \int_0^1 \frac{x^2}{\sqrt{x^2 + 2}} dx \quad , \quad K = \int_0^1 \sqrt{x^2 + 2} dx$$

$$f(x) = \ln(x + \sqrt{x^2 + 2}) \quad (1)$$

$$\forall x \in \mathbb{R} \quad f'(x) = \frac{1 + \frac{x}{\sqrt{x^2 + 2}}}{x + \sqrt{x^2 + 2}} \quad - a$$

$$= \frac{x + \sqrt{x^2 + 2}}{\sqrt{x^2 + 2} (x + \sqrt{x^2 + 2})}$$

$$\boxed{f'(x) = \frac{1}{\sqrt{x^2 + 2}}} \quad \text{إذن :}$$

$$I = \int_0^1 \frac{dx}{\sqrt{x^2 + 2}} \quad - b$$

$$= \left[\ln(x + \sqrt{x^2 + 2}) \right]_0^1$$

$$\boxed{I = \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right)} \quad \text{إذن :}$$

$$J + 2I = \int_0^1 \left(\frac{x^2}{\sqrt{x^2 + 2}} + \frac{2}{\sqrt{x^2 + 2}} \right) dx \quad (2)$$

$$= \int_0^1 \frac{x^2 + 2}{\sqrt{x^2 + 2}} dx$$

$$= \int_0^1 \sqrt{x^2 + 2} dx$$

$$\boxed{J + 2I = K} \quad \text{إذن :}$$

$$(3) \text{ نضع : } \begin{cases} u'(x) = \frac{x}{\sqrt{x^2 + 2}} & , & \begin{cases} u(x) = \sqrt{x^2 + 2} \\ v'(x) = 1 \end{cases} \\ v(x) = x \end{cases}$$

$$K = \left[x \sqrt{x^2 + 2} \right]_0^1 - \int_0^1 \frac{x^2}{\sqrt{x^2 + 2}} dx$$

$$= \sqrt{3} - J$$

$$\boxed{K = \sqrt{3} - J} \quad \text{إذن :}$$

(4) بما أن :

$$I = \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right) , \quad J + 2I = K , \quad K = \sqrt{3} - J$$

فإن :

$$I = \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right) , \quad J + 2I = \sqrt{3} - J , \quad K = \sqrt{3} - J$$

ومنه فإن :

$$I = \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right) , \quad J = \frac{1}{2}(\sqrt{3} - 2I) , \quad K = \sqrt{3} - J$$

إذن :

$$\boxed{J = \frac{\sqrt{3}}{2} - \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right)} \quad \text{و} \quad \boxed{K = \frac{\sqrt{3}}{2} + \ln\left(\frac{1 + \sqrt{3}}{\sqrt{2}}\right)}$$

إعداد الأستاذ : محمد أنفلوس