

$$u_n = \int_0^{\frac{1}{2}} \frac{dt}{\sqrt{1-t^n}} \quad (1)$$

$$0 \leq t \leq \frac{1}{2} \Rightarrow 0 \leq t^n \leq \frac{1}{2^n} \quad ; n \in \mathbb{N}^*$$

$$\Rightarrow 1 - \frac{1}{2^n} \leq 1 - t^n \leq 1 \quad ; n \in \mathbb{N}^*$$

$$\Rightarrow \sqrt{1 - \frac{1}{2^n}} \leq \sqrt{1 - t^n} \leq 1 \quad ; n \in \mathbb{N}^*$$

$$\Rightarrow 1 \leq \frac{1}{\sqrt{1 - t^n}} \leq \frac{1}{\sqrt{1 - \frac{1}{2^n}}} \quad ; n \in \mathbb{N}^*$$

$$\Rightarrow \int_0^{\frac{1}{2}} 1 dt \leq \int_0^{\frac{1}{2}} \frac{dt}{\sqrt{1 - t^n}} \leq \int_0^{\frac{1}{2}} \frac{dt}{\sqrt{1 - \frac{1}{2^n}}}$$

$$\Rightarrow [t]_0^{\frac{1}{2}} \leq u_n \leq \frac{1}{\sqrt{1 - \frac{1}{2^n}}} [t]_0^{\frac{1}{2}}$$

$$\boxed{\forall n \in \mathbb{N}^* \quad \frac{1}{2} \leq u_n \leq \frac{1}{2\sqrt{1 - \frac{1}{2^n}}}} \quad \text{إذن :}$$

$$0 < \frac{1}{2} < 1 \Rightarrow \lim_{n \rightarrow +\infty} \left(\frac{1}{2}\right)^n = 0$$

$$\Rightarrow \lim_{n \rightarrow +\infty} \frac{1}{2\sqrt{1 - \left(\frac{1}{2}\right)^n}} = \frac{1}{2}$$

$$\Rightarrow \boxed{\lim_{n \rightarrow +\infty} u_n = \frac{1}{2}}$$

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