

$$\begin{aligned}
 2 (\cos \theta - 1) + \frac{1}{1 + \cos \theta} &= \frac{2 (1 + \cos \theta) (\cos \theta - 1) + 1}{1 + \cos \theta} \\
 &= \frac{2 (\cos^2 \theta - 1) + 1}{1 + \cos \theta} \\
 &= \frac{2 \cos^2 \theta - 1}{1 + \cos \theta}
 \end{aligned}$$

إذن :

$$\forall \theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4} \right] \quad 2 (\cos \theta - 1) + \frac{1}{1 + \cos \theta} = \frac{\cos 2 \theta}{1 + \cos \theta}$$

ب - نعلم أن $\cos (2 \alpha) = 2 \cos^2 \alpha - 1$

$$1 + \cos \theta = 2 \cos^2 \left(\frac{\theta}{2} \right)$$

إذن :

$$\begin{aligned}
 \frac{\sin (2 x)}{1 + \cos \left(x - \frac{\pi}{4} \right)} &= \frac{\cos \left(\frac{\pi}{2} - 2 x \right)}{1 + \cos \left(x - \frac{\pi}{4} \right)} \quad (2) \text{ لدينا :} \\
 &= \frac{\cos \left[2 \left(x - \frac{\pi}{4} \right) \right]}{1 + \cos \left(x - \frac{\pi}{4} \right)} \\
 &= 2 \left[\cos \left(x - \frac{\pi}{4} \right) - 1 \right] + \frac{1}{1 + \cos \left(x - \frac{\pi}{4} \right)} \\
 &= 2 \left[\cos \left(x - \frac{\pi}{4} \right) - 1 \right] + \frac{1}{2 \cos^2 \left(\frac{x - \frac{\pi}{4}}{2} \right)}
 \end{aligned}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin (2 x)}{1 + \cos \left(x - \frac{\pi}{4} \right)} dx \quad \text{إذن :}$$

$$\begin{aligned}
 &= \int_0^{\frac{\pi}{2}} \left[2 \left(\cos \left(x - \frac{\pi}{4} \right) - 1 \right) + \frac{\frac{1}{2}}{\cos^2 \left(\frac{x}{2} - \frac{\pi}{8} \right)} \right] dx \\
 &= \left[2 \sin \left(x - \frac{\pi}{4} \right) - 2 x + \tan \left(\frac{x}{2} - \frac{\pi}{4} \right) \right]_0^{\frac{\pi}{2}}
 \end{aligned}$$

$$I = 2 \sqrt{2} + 1 - \pi$$

إذن :

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