

$$(S) : x^2 + y^2 + z^2 + 6x - 8y + 2z + 17 = 0 \quad (1)$$

$$(D) : \begin{cases} x = -1 + 2t \\ y = -t \\ z = t \end{cases} \quad (t \in \mathbb{R})$$

$$M(x, y, z) \in (S) \cap (D)$$

$$\begin{aligned} \Leftrightarrow & \begin{cases} x = -1 + 2t \\ y = -t \\ z = t \\ x^2 + y^2 + z^2 + 6x - 8y + 2z + 17 = 0 \end{cases} \\ \Leftrightarrow & \begin{cases} x = -1 + 2t \\ y = -t \\ z = t \\ (-1 + 2t)^2 + (-1)^2 + t^2 + 6(-1 + 2t) - 8(-t) + 2t + 17 = 0 \end{cases} \\ \Leftrightarrow & \begin{cases} x = -1 + 2t \\ y = -t \\ z = t \\ 6(t^2 + 3t + 2) = 0 \end{cases} \\ \Leftrightarrow & \begin{cases} x = -1 + 2t \\ y = -t \\ z = t \\ (t + 1)(t + 2) = 0 \end{cases} \\ \Leftrightarrow & \begin{cases} t = -1 + 2t \\ x = -3 \\ y = 1 \\ z = -1 \end{cases} \quad \text{أو} \quad \begin{cases} t = -2 \\ x = -5 \\ y = 2 \\ z = -2 \end{cases} \end{aligned}$$

$$(D) \cap (S) = \{A(-3, 1, -1) ; B(-5, 2, -2)\}$$

إذن

$$(S) : x^2 + y^2 + z^2 - 2x + 2z + 1 = 0 \quad (2)$$

$$(D) : \begin{cases} x = t \\ y = 1 - 2t \\ z = -1 + 2t \end{cases} \quad (t \in \mathbb{R})$$

$$M(x, y, z) \in (S) \cap (D)$$

$$\Leftrightarrow \begin{cases} x = t \\ y = 1 - 2t \\ z = -1 + 2t \\ x^2 + y^2 + z^2 - 2x + 2z + 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = t \\ y = 1 - 2t \\ z = -1 + 2t \\ t^2 + (1 - 2t)^2 + (-1 + 2t)^2 - 2t + 2(-1 + 2t) + 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = t \\ y = 1 - 2t \\ z = -1 + 2t \\ 9t^2 + 6t + 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = t \\ y = 1 - 2t \\ z = -1 + 2t \\ (3t + 1)^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = t \\ y = 1 - 2t \\ z = -1 + 2t \\ t = -\frac{1}{3} \end{cases}$$

$$(S) \cap (D) = \left\{ E\left(-\frac{1}{3}, \frac{5}{3}, \frac{-5}{3}\right) \right\} \quad \text{إذن}$$

$$(S) : x^2 + y^2 + z^2 - 4x + 1 = 0 \quad (3)$$

$$(D) : \begin{cases} x = 2t \\ y = 1 + t \\ z = t \end{cases} \quad (t \in \mathbb{R})$$

$$M(x, y, z) \in (S) \cap (D)$$

$$\Leftrightarrow \begin{cases} x = 2t \\ y = 1 + t \\ z = t \\ x^2 + y^2 + z^2 - 4x + 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2t \\ y = 1 + t \\ z = t \\ (2t)^2 + (1 + t)^2 + t^2 - 4(2t) + 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2t \\ y = 1 + t \\ z = t \\ 2(3t^2 - 3t + 1) = 0 \end{cases}$$

بما أن المعادلة $3t^2 - 3t + 1 = 0$ لا تقبل أي حل في $\mathbb{R}$ فإن المستقيم (D) لا تقطع الفلكة (S).

إعداد الأستاذ : عبد العزيز فرحاني